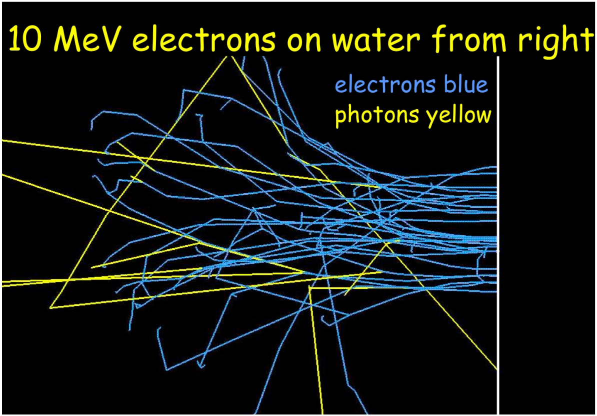
PHYS 5012
Radiation Physics and Dosimetry
Lecture 5

Tuesday 31 March 2009

Charged Particle Interactions (cont.)

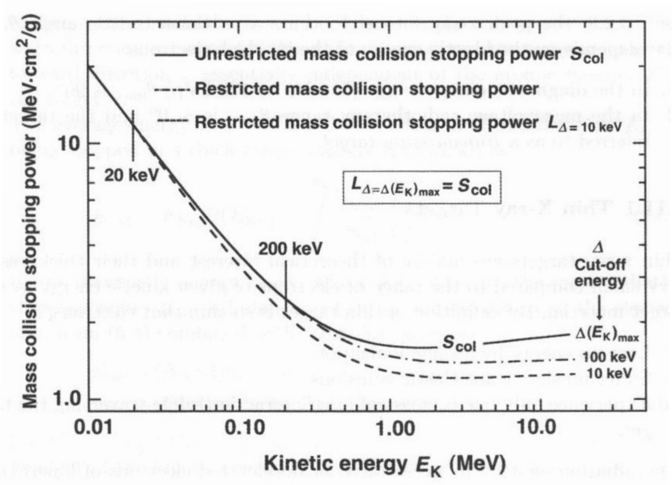


Stopping power of water for $10\,\mathrm{MeV}$ electrons:
$S_{\mathrm{col}} = 1.968\,\mathrm{MeV\,cm^2\,g^{-1}}$, $S_{\mathrm{rad}} = 0.1814\,\mathrm{MeV\,cm^2\,g^{-1}}$,
$S_{\mathrm{tot}} = S_{\mathrm{col}} + S_{\mathrm{rad}} = 2.149\,\mathrm{MeV\,cm^2\,g^{-1}}$
Range: $R_{\mathrm{CSDA}} = 4.975\,\mathrm{g\,cm^{-2}} \Rightarrow$ average path length of $10\,\mathrm{MeV}$ electrons in water is $\approx 5\,\mathrm{cm}$.

Restricted Stopping Power

- S_{col} includes both hard and soft collisions; hard collisions can produce secondary electrons (δ rays) carrying significant kinetic energy away from primary particle track $\Rightarrow$ non-localised energy deposition
- localised energy transfer measured by excluding δ rays above a threshold energy Δ
- $\Delta < \Delta E_{\mathrm{max}} =$ maximum energy transfer to δ rays
- $L_{\Delta} =$ *restricted* mass collision stopping power $< S_{\mathrm{col}}$
- $L_{\Delta} = S_{\mathrm{col}}$ when $\Delta = \Delta E_{\mathrm{max}}$
- for nonrelativistic heavy charged particles, $\Delta E_{\mathrm{max}} \approx 2 m_{\mathrm{e}} \beta^2 c^2$ (c.f. Lec. 4 eqn. 7)

Example: What is the energy of a proton than can produce a δ-ray with enough energy to traverse a cell with a diameter $2.5\,\mu\text{m}$? From the NIST/estar database, the energy of an electron with $R_{\text{CSDA}}/\rho = 2.5 \times 10^{-4}\,\text{cm}$ is $10\,\text{keV}$ (assuming the cell tissue is water-equivalent). Equating this to $E_{\text{max}} = 2m_{\text{e}}\beta^2c^2$ gives $\beta^2 = 0.01$ and hence, $(\gamma-1)m_{\text{p}}c^2 = 4.7\,\text{MeV}$. A proton with more energy than this would produce more energetic δ-rays that could deposit their energy on scales larger than the cell size, thus overshooting the irradiation target. In this case, a restricted stopping power of $L_{10\,\text{keV}}$ for electrons in water would be appropriate.

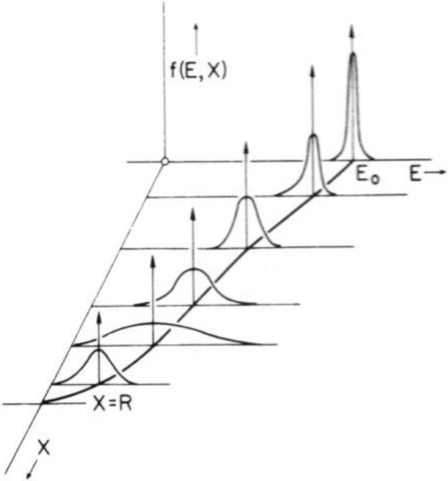


Unrestricted (solid curves) and restricted collision stopping powers, with $\Delta = 10\,\text{keV}$ and $\Delta = 100\,\text{keV}$ (dashed curves) for electrons in carbon. (Fig. 5.9 in Podgoršak.)

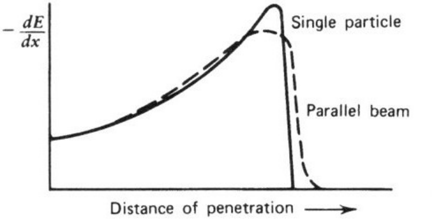
- for incident electrons, $\Delta E_{\text{max}} = \frac{1}{2} E_{\text{K}}$, so $L_\Delta = S_{\text{col}}$ when $2\Delta = E_{\text{K}}$ (see above figure)

Straggling and Scattering

- energy straggling is the formation of a distribution of particle energies resulting from the stochastic nature of energy loss interactions in a medium



- range straggling is the formation of a distribution of pathlengths traversed by particles in a medium before they stop; it also results from stochastic changes in rate of energy loss



- multiple Coulomb scattering results in a spread of pathlengths and dispersion of an initially parallel beam ("pencil beam") of charged particle into a diverging 3D conical beam

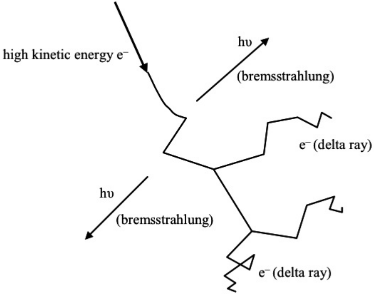
- many collisions with large b, but small θ
- fewer collisions with small b, but large θ
- symmetric dispersion perpendicular to beam

$$\theta_{\text{rms}}^2 = \frac{1}{2}\int \theta^2 N l \frac{d\sigma}{d\Omega}\, d\Omega \qquad \text{rms angular spread} \tag{1}$$

where $N =$ density of target material, $l =$ thickness of material.

Electron Range

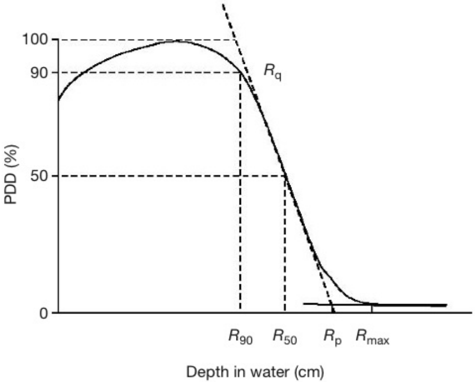
- electrons suffer large deviations in their trajectories as a result of elastic and inelastic scattering $\rightarrow$ haphazard electron tracks, with secondary δ rays



Life history of a fast electron

- lower energy electrons suffer more scatterings and larger deviations
- scattering increases with Z ($R_{\mathrm{CSDA}} \propto Z$ at low E_{K})

- maximum penetration depth, R_{max} (in units $\mathrm{kg\,m^{-2}}$), can be $\ll R_{\mathrm{CSDA}}$, especially for low E_{K} electrons
- $R_{\mathrm{max}}/R_{\mathrm{CSDA}} \simeq 0.5$ for high-Z; e.g. for $1\,\mathrm{MeV}$ electrons: in Pb, $R_{\mathrm{max}}/R_{\mathrm{CSDA}} \simeq 0.57$; in C, $R_{\mathrm{max}}/R_{\mathrm{CSDA}} \simeq 0.95$
- R_{max} measured from percentage depth-dose curve:



Typical Percentage Depth Dose (PDD) curve for an arbitrary electron beam in water. R_{50} is the mean range, R_{p} is the projected range

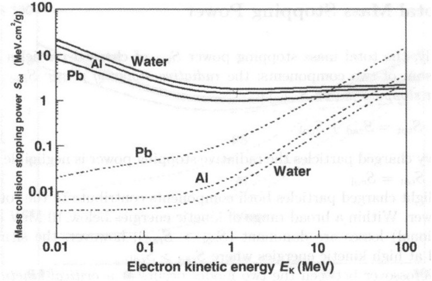
Energy Deposition

- absorbed dose is the energy deposited by charged particles in a medium per unit mass of the medium:

$$D = \frac{\Delta E_{\mathrm{ab}}}{\Delta m} = \Phi \left(\frac{-1}{\rho} \frac{dE}{dx} \right) = \Phi \, S_{\mathrm{col}} \tag{2}$$

where $\Phi =$ fluence (no. particles per unit area).

Recall from Lec. 4, S_{col} vs. E_{K} for electrons:

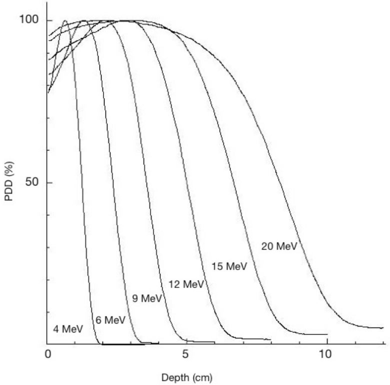


Mass collision stopping power (solid curves) and radiative stopping power (dashed curves) for electrons. (Fig. 5.5 in Podgoršak.)

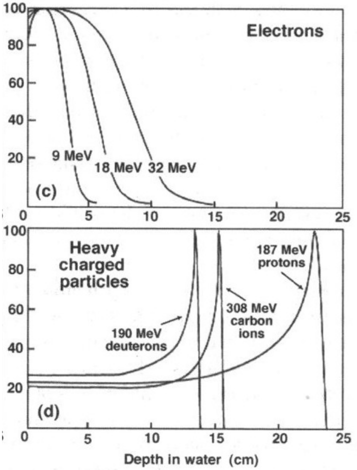
- S_{col} for electrons is smoothly varying w.r.t. E_{K}

$\Rightarrow$ as incident electrons with $E_{\text{K}} \gg m_{\text{e}}c^2$ lose their energy, dose remains relatively steady, with a gradual build up due to increase number of electrons from ionisations, then a gradual drop in D as E_{K} drops below $m_{\text{e}}c^2$ and S_{col} increases

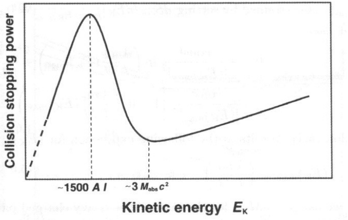
- R_{max} decreases as S_{col} increases, so dose deposited over shorter depth for lower $E_{\text{K},0}$



- compare electron $R_{\max}$, $S_{\rm col}$ and depth dose curves to those for heavy charged particles:



The localised deposition of energy ('Bragg peak" in depth-dose curve) for heavy charged particles results from the sharp increase in $S_{\rm col}$ as the particles become nonrelativistic. For the same $R_{\max}$, electrons deposit their energy throughout most of the depth traversed.



Linear Energy Transfer

- Bethe formula for S_{col} for heavy charged particles (c.f. Lec. 4) $\Rightarrow$

$$\frac{dE}{dx} \propto \rho \frac{z^2}{\beta^2}\left[\ln\left(\frac{2m_e c^2\gamma^2\beta^2}{I}\right) - \beta^2\right] \tag{3}$$

- $\lim \beta \ll 1$ (Bethe-Bloch formula):

$$\frac{dE}{dx} \propto \rho \frac{z^2}{\beta^2}\left[\ln\left(\frac{2m_e v^2}{I}\right)\right]$$

- dE/dx for light charged particles has similar β^{-2} dependence and somewhat weaker logarithmic dependence; but, β is higher for a given E_K
- heavy charged particles have a higher linear energy transfer (LET) – energy transfer is more localised

Bremsstrahlung (Radiation) Yield

- $B(E_{\mathrm{K},0}) =$ fraction of initial kinetic energy $E_{\mathrm{K},0}$ emitted as bremsstrahlung radiation through continuous slowing down of charged particle in a medium
- $B(E_{\mathrm{K},0}) = 0$ for heavy charged particles
- for light charged particles (electrons and positrons),

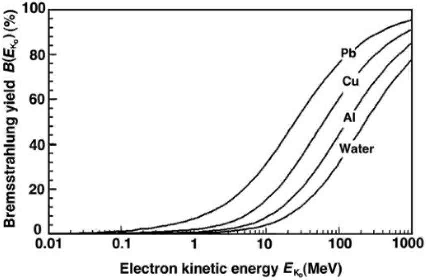
$$B(E_{\mathrm{K},0}) = \frac{1}{E_{\mathrm{K},0}} \int_0^{E_{\mathrm{K},0}} \frac{S_{\mathrm{rad}}(E)}{S_{\mathrm{tot}}(E)}\, dE \qquad (4)$$

- electron-positron annilhilation also produces radiation, but is negligible compared to bremsstrahlung emission
- energy radiated per charged particle:

$$E_{\mathrm{rad}} = E_{\mathrm{K},0} B(E_{\mathrm{K},0}) = \int_0^{E_{\mathrm{K},0}} \frac{S_{\mathrm{rad}}(E)}{S_{\mathrm{tot}}(E)}\, dE \qquad (5)$$

Recall from Lec. 4 (c.f. eqn. 10), Bethe-Heitler formula:

$$S_{\text{rad}} \propto \frac{Z^2}{A} B_{\text{rad}}(Z, E_{\text{i}})\, E_{\text{i}} \qquad , \qquad E_{\text{i}} = E_{\text{K},0} + m_{\text{e}} c^2$$



Bremsstrahlung yield for electrons in different media plotted against initial electron kinetic energy. (Fig. 5.7 in Podgoršak.)

- energy lost to ionisation per charged particle:

$$E_{\text{col}} = E_{\text{K},0} - E_{\text{rad}} = E_{\text{K},0}\left[1 - B(E_{\text{K},0})\right] = \int_0^{E_{\text{K},0}} \frac{S_{\text{col}}(E)}{S_{\text{tot}}(E)}\, dE \tag{6}$$

Example: Radiation yield for $10\,\mathrm{MeV}$ electrons in water $\approx 4.1\%$ (from NIST/estar database) $\Longrightarrow$

- energy radiated per charged particle:

$$E_{\mathrm{rad}} = E_{\mathrm{K},0} B(E_{\mathrm{K},0}) \approx 0.41\,\mathrm{MeV}$$

- energy lost to ionisation per charged particle:

$$E_{\mathrm{col}} = E_{\mathrm{K},0}\left[1 - B(E_{\mathrm{K},0})\right] \approx 9.59\,\mathrm{MeV}$$

Radiation Yield and Radiative Fraction

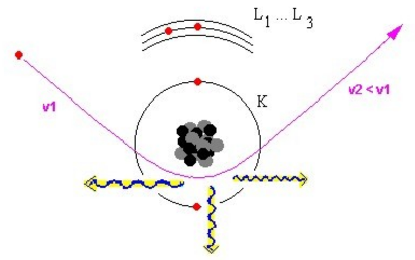
- $B(E_{\mathrm{K},0}) =$ radiation yield = fraction of initial kinetic energy $E_{\mathrm{K},0}$ of an electron (or positron) lost to radiative (bremsstrahlung) losses
- $\bar{g} =$ radiative fraction = *average* fraction of energy transferred to light charged particles by photons that is subsequently lost to radiation (predominantly bremsstrahlung); c.f.

$$\frac{\mu_{\mathrm{ab}}}{\rho} = \frac{\mu_{\mathrm{tr}}}{\rho}\left(1 - \bar{g}\right)$$

- photon interactions produce electrons (and positrons) with a *spectrum* of kinetic energies $E_{\mathrm{K,i}} \Rightarrow \bar{g} =$ *average* value of $B(E_{\mathrm{K,i}})$ for all light charged particles produced by photon interactions

Bremsstrahlung Targets

- monoenergetic electron beams incident on solid target material commonly used to generate (bremsstrahlung) radiation beams



Production of Bremsstrahlung

- thin X-ray targets – thickness $\Delta x \ll R$ sufficiently small such that incident electrons experience
  - no ionisation losses
  - no elastic collisions
  - only one bremsstrahlung interaction
- total energy radiated:

$$(E_{\mathrm{rad}})_{\mathrm{tot}} = \rho N_{\mathrm{e}} \Delta x \, S_{\mathrm{rad}} = N_{\mathrm{e}} \Delta x \left(- \frac{dE_{\mathrm{rad}}}{dx} \right)$$

where $N_{\mathrm{e}} =$ no. of incident electrons

- intensity spectrum, I_E (where $E = h\nu$), obtained by taking Fourier transform of classical Larmor formula (c.f. Lec. 1) for power emitted in electromagnetic radiation
  - $\rightarrow dI(b)/d\omega \propto (vb)^{-2} \Rightarrow$ fewer photons emitted for small-b interactions of monoenergetic electrons
  - $h\nu \propto b^{-1}$
  - integrate over all impact parameters: $I_E \propto \int (dI(b)/d\omega)\, b db \propto \ln(b_{\max}/b_{\min}) \rightarrow$ approx. independent of $h\nu$

  $\Rightarrow$ *thin-target bremsstrahlung spectrum is approximately flat* up to a cut-off $h\nu_{\max} = E_{K,0}$

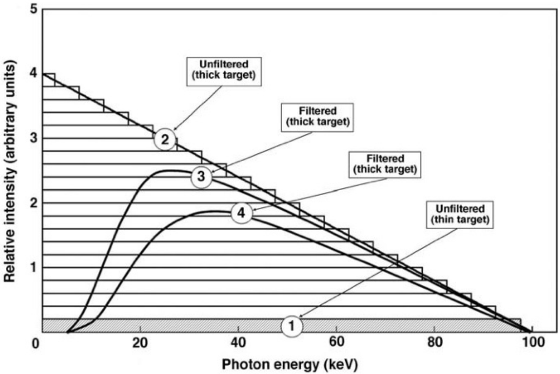
Thick Targets

- $\Delta x \sim R$ such that
  - no incident electrons can traverse the medium
  - attenuation of bremsstrahlung photons is minimised
- intensity spectrum is a superposition of multiple thin-target spectra for different $E_{\mathrm{K,i}}$, since incident monoenergetic electrons gradually lose energy through multiple collisions

$$I_E = CZ\left(E_{\mathrm{K},0} - h\nu\right) \qquad \text{Kramer's spectrum} \qquad (7)$$

where $C =$ constant and $E_{\mathrm{K},0} =$ initial kinetic energy of electrons incident on thick target

- filtration of resulting X-ray beam through attenuators preferentially removes low enegy photons and *hardens* the spectrum; used for clinical applications



Typical bremsstrahlung spectra produced by $100\,\mathrm{keV}$ electrons striking a target: 1. thin target; 2. thick target (unfiltered) showing superposition of multiple thin target spectra; 3. thick target, with resulting beam filtered by an X-ray tube window; 4. thick target with additional filtration. In general, spectra should also include characteristic emission from the target and filters.

- for $E_{\mathrm{K},0} \ll m_e c^2$,

$$S_{\mathrm{rad}} = \alpha r_e^2 Z^2 \frac{N_A}{A} B_{\mathrm{rad}} (E_{\mathrm{K},0} + m_e c^2) \propto Z^2 \frac{N_A}{A} B_{\mathrm{rad}}$$

⇒ energy radiated:

$$E_{\mathrm{rad}} = \int_0^{E_{\mathrm{K},0}} \frac{S_{\mathrm{rad}}(E)}{S_{\mathrm{tot}}(E)} dE \approx S_{\mathrm{rad}} \int_0^{E_{\mathrm{K},0}} \frac{dE}{S_{\mathrm{tot}}(E)}$$

and $S_{\mathrm{tot}} \approx S_{\mathrm{col}} \propto Z N_A / A \Rightarrow$

$$E_{\mathrm{rad}} \propto Z$$

⇒ high-Z targets are more efficient for bremsstrahlung production of *kilovoltage* X-ray beams

- for $E_{\mathrm{K},0} \gg m_e c^2$,

$$\frac{S_{\mathrm{rad}}}{S_{\mathrm{tot}}} = \frac{S_{\mathrm{rad}}}{(S_{\mathrm{col}} + S_{\mathrm{rad}})} = \frac{E_{\mathrm{K},0}}{(n/Z + E_{\mathrm{K},0})}$$

  where $n = n(Z, E_{\mathrm{K},0})$ is a weak function of Z and $E_{\mathrm{K},0}$

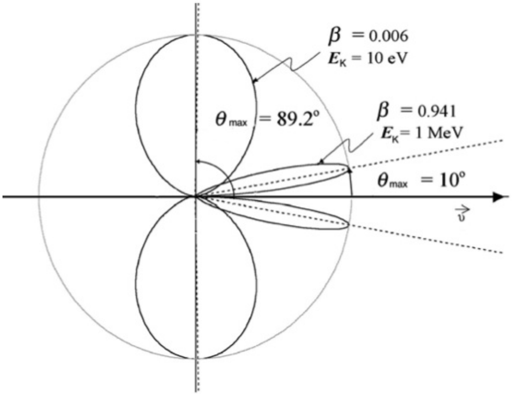
$\Rightarrow$ production of *megavoltage* X-ray beams largely independent of target Z

- Bethe-Heitler quantum-mechanical differential cross-section for photon emission by relativistic electrons predicts

$$\frac{d\sigma_{\mathrm{rad}}}{d(h\nu)} \propto \frac{B_{\mathrm{rad}}}{h\nu} \Rightarrow I_E \propto B_{\mathrm{rad}} E^{-1}$$

  where B_{rad} is a slowly decreasing function of $E = h\nu$

radiation pattern



- for $E_K \ll m_e c^2$, most radiation emerges at $\theta_{max} \approx 90°$
- for $E_K \gg m_e c^2$, most radiation emerges at $\theta_{max} \simeq 0°$ (forward beaming)

This is why X-ray tubes, which produce kilovoltage X-rays, have an exit window positioned at an angle to the target, while medical linacs, which produce megavoltage X-rays, have an X-ray window placed on the other side of the target: