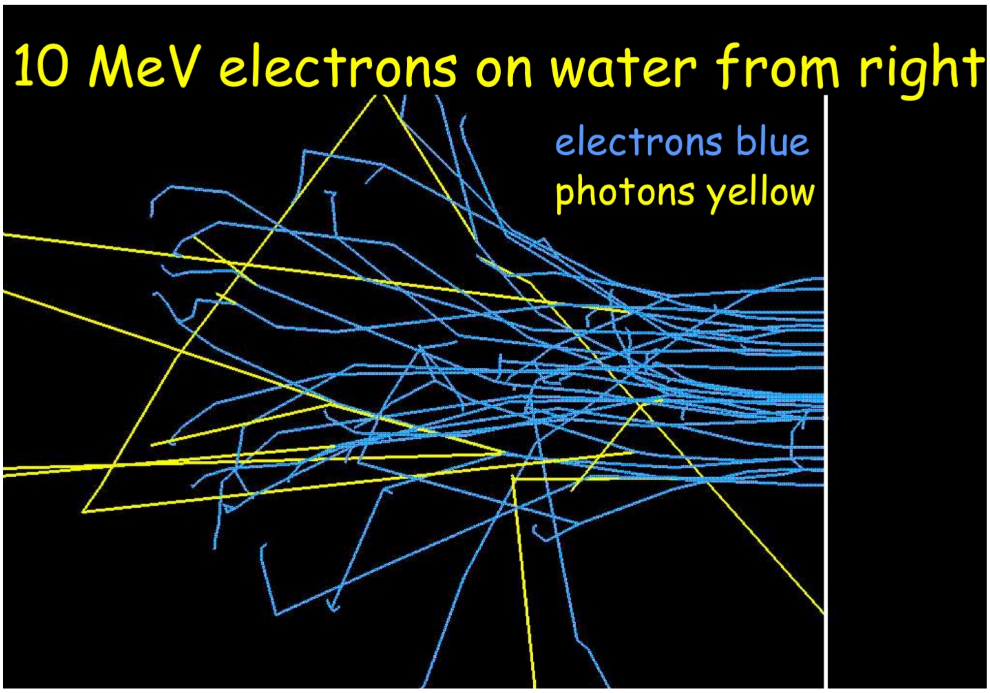
PHYS 5012
Radiation Physics and Dosimetry

Lecture 4

Tuesday 24 March 2009

Contents

1 Interactions of Charged Particles with Matter



1.1 General Aspects

Collisions between two particles involve a *projectile* and a *target*.

Types of targets: whole atoms, atomic nuclei, atomic orbital electrons.

Types of projectiles:

- heavy charged particles (protons, α-particles, heavy ions)
- light charged particles (electrons, positrons)
- neutrons (not considered here)

3 categories:

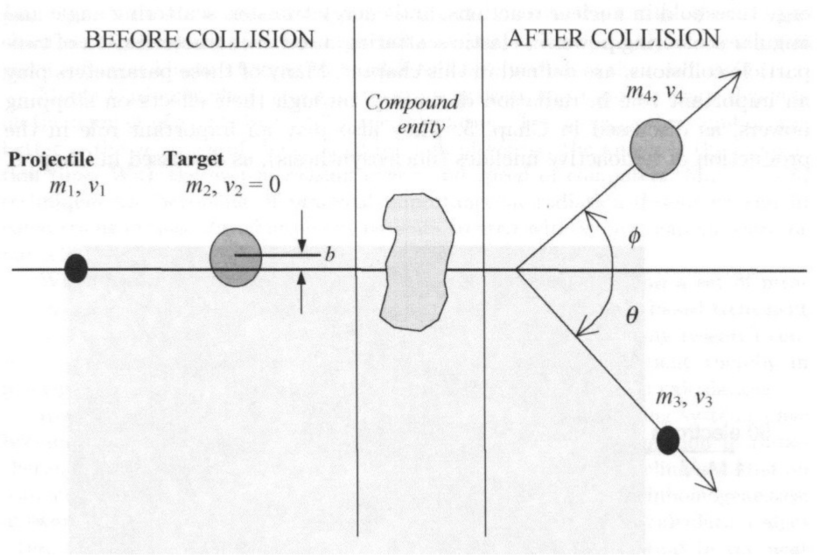
1. *Nuclear reactions* – final reaction products differ from initial particles; charge, momentum and mass-energy conserved; e.g. deuteron bombarding nitrogen-14: ${}^{14}_{7}N(d,p){}^{15}_{7}N$
2. *Elastic collisions* – final products identical to initial particles; kinetic energy and momentum conserved; e.g. Rutherford scattering of α particle on gold nucleus: ${}^{197}_{79}Au(\alpha,\alpha){}^{197}_{79}Au$

3. *Inelastic collisions* – final products identical to initial particles; kinetic energy *not* conserved

In inelastic collisions, some kinetic energy is converted to *excitation energy* in the form of:

- *nuclear excitation* of target resulting from heavy charged particle striking target nucleus; e.g. ${}^{\mathrm{A}}_{\mathrm{Z}}X(\alpha,\alpha){}^{\mathrm{A}}_{\mathrm{Z}}X^*$
- *atomic excitation or ionisation* of target resulting from heavy or light charged particle colliding with target orbital electron
- *bremsstrahlung emission* by light charged particle projectile resulting from Coulomb interaction with target nucleus

1.1.1 Nuclear Reactions



Schematic illustration of a general nuclear reaction. (Fig. 4.1 in Podgoršak.)

- intermediate compound produced temporarily; spontaneously decays into *reaction products*
- conservation of atomic number: $\sum Z_{\text{before}} = \sum Z_{\text{after}}$
- conservation of atomic mass: $\sum A_{\text{before}} = \sum A_{\text{after}}$

Conservation of momentum

$$\mathbf{p}_1 = \mathbf{p}_3 + \mathbf{p}_4 \tag{1}$$

$$\Longrightarrow \begin{array}{ll} p_1 = p_3\cos\theta + p_4\cos\phi & \parallel \text{ to } \mathbf{p}_1 \\ 0 = p_3\sin\theta + p_4\sin\phi & \perp \text{ to } \mathbf{p}_1 \end{array}$$

Conservation of mass-energy

$$\big(m_1c^2 + E_{\mathrm{K},1}\big) + m_2c^2 = \big(m_3c^2 + E_{\mathrm{K},3}\big) + \big(m_4c^2 + E_{\mathrm{K},4}\big) \tag{2}$$

where $E_{\mathrm{K}} =$ particle kinetic energy $= (\gamma - 1)mc^2$

$$Q = \big(m_1c^2 + m_2c^2\big) - \big(m_3c^2 + m_4c^2\big) \qquad \textit{Q value} \tag{3}$$

- $Q > 0 \Rightarrow$ *exothermic* collision
- $Q = 0 \Rightarrow$ *elastic* collision
- $Q < 0 \Rightarrow$ *endothermic* collision

Threshold Energy

- minimum projectile energy E_{thr} required for endothermic reaction to proceed

Conservation of 4-momentum, $p = (E/c, \mathbf{p})$:

$$p_1 + p_2 = p_3 + p_4 \Rightarrow (p_1 + p_2)^2 = (p_3 + p_4)^2$$

and using $p_1^2 = (E_1/c)^2 - |\mathbf{p}_1|^2 = m_1^2c^2$ and $p_2^2 = (E_2/c)^2 = m_2^2c^2$, gives

$$2E_1E_2 = (p_3 + p_4)^2c^2 - (m_1^2c^4 + m_2^2c^4)$$

Note that $p_3 + p_4$ is the centre-of-mass 4-momentum, p_{cm}, and so $(p_3 + p_4)^2 = p_{\text{cm}}^2 = (E_{\text{cm}}/c)^2 = (m_3c^2 + m_4c^2)^2/c^2$ since the modulus of a 4-vector is invariant and has the same value in any frame of reference. So the threshold energy E_1 for the projectile is:

$$E_{\text{thr}} = \frac{(m_3c^2 + m_4c^2)^2 - (m_1^2c^4 + m_2^2c^4)}{2m_2c^2} \tag{4}$$

corresponding *threshold kinetic energy*:

$$E_{\mathrm{K,thr}} = \frac{(m_3c^2 + m_4c^2)^2 - (m_1c^2 + m_2c^2)^2}{2m_2c^2} \tag{5}$$

in terms of the Q value:

$$\begin{aligned} E_{\mathrm{K,thr}} &= -Q\left[\frac{m_1c^2 + m_2c^2}{m_2c^2} - \frac{Q}{2m_2c^2}\right] \\ &\approx -Q\left(1 + \frac{m_1}{m_2}\right) \end{aligned} \tag{6}$$

if $Q \ll m_2 c^2$ (usually the case).

The Q value is defined for general two-particle collisions.

For pair production, for example, $m_1 = 0$, $m_2 = m_3 \gg m_{\rm e}$ and $Q = -2m_{\rm e}c^2$, so

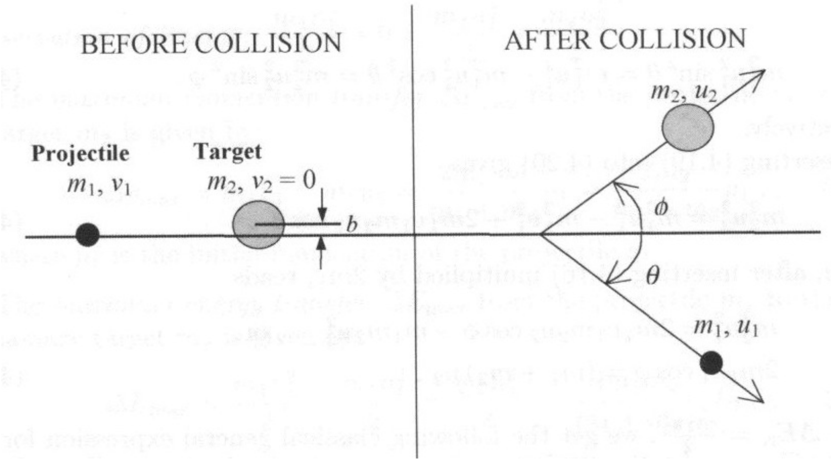
$$\left(E_\gamma^{\rm pp}\right)_{\rm thr} = 2m_{\rm e}c^2$$

while for triplet production, $Q = -2m_{\rm e}c^2$ but $m_2 = m_{\rm e}$, so

$$\left(E_\gamma^{\rm tp}\right)_{\rm thr} = 4m_{\rm e}c^2$$

1.1.2 Elastic Scattering

- initial and final particles remain the same (i.e. $m_3 = m_1$ and $m_4 = m_2$), so $Q = 0$
- kinetic energy transfer $\Delta E_{\rm K}$ from m_1 to m_2



Schematic illustration of elastic scattering. θ is the scattering angle, ϕ is the recoil angle and b is the impact parameter. (Fig. 4.2 in Podgoršak.)

Classical derivation of *kinetic energy transfer*: conservation of momentum and energy ⇒

$$\Delta E_{\rm K} = \frac{1}{2}m_2 u_2^2 = E_{\rm K1}\frac{4m_1 m_2}{(m_1+m_2)^2}\,\cos^2\phi \tag{7}$$

Head-on collisions:

- $b = 0$ and $\phi = 0$
- maximum energy and momentum transfer
- $\theta = 0$ (forward scattering) when $m_1 > m_2$
- $\theta = \pi$ (back-scattering) when $m_1 < m_2$

- projectile stops when $m_1 = m_2$

Example: proton colliding with orbital electron Maximum energy transfer (for a head-on collision), noting that $m_\mathrm{p} \gg m_\mathrm{e}$:

$$\Delta E_\mathrm{max} \approx 4 E_\mathrm{Kp} \frac{m_\mathrm{e}}{m_\mathrm{p}} \approx 2 \times 10^{-3} E_\mathrm{Kp}$$

Collisions between particles of the same mass ($m_1 = m_2$):

- *distinguishable particles* (e.g. electron colliding with positron): $\Delta E_\mathrm{max} = E_\mathrm{K1}$ $\Rightarrow$ head-on collision transfers *all* projectile's kinetic energy to target
- *indistinguishable particles* (e.g. free electron colliding with bound electron): $\Delta E_\mathrm{max} = \frac{1}{2} E_\mathrm{K1}$

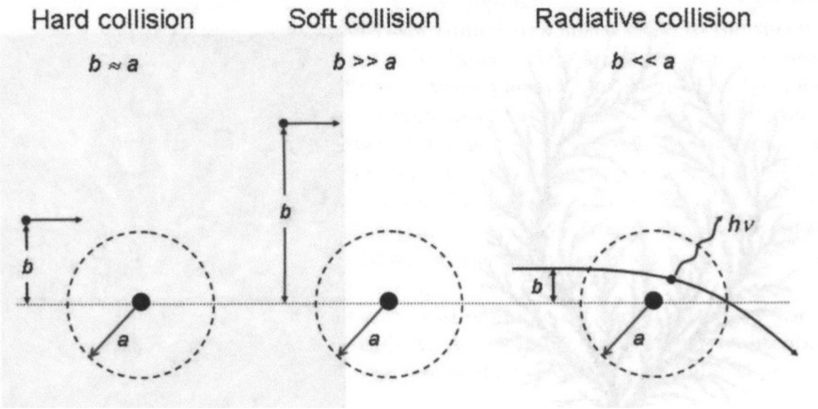
Relativistic formula for energy transfer in a head-on collision:

$$\Delta E_\mathrm{max} = (\gamma_2 - 1) m_2 c^2 = \frac{2(\gamma + 1) m_1 m_2}{m_1^2 + m_2^2 + 2\gamma m_1 m_2} E_\mathrm{K1} \tag{8}$$

where $\gamma m_1 c^2 =$ energy of incident projectile

1.2 Stopping Power

- Stopping power measures ability of matter to stop charged particles
- incident charged particle loses all kinetic energy via multiple Coulomb interactions (mostly elastic, but sometimes inelastic)
- gradual loss of kinetic energy called *continuous slowing down approximation* (CSDA)
- e.g. 1 MeV charged particle typically undergoes $\sim 10^5$ interactions before losing all its kinetic energy
- stochastic process $\Rightarrow$ need to use probabilities and average quantities
- *hard collisions*: Coulomb interactions with orbital electron for $b \approx a$
- *soft collisions*: Coulomb interaction with orbital electron for $b \gg a$
- *radiative collisions*: Coulomb interactions with nuclear field for $b \ll a$



The three different types of collisions depend on the classical impact paramater b and atomic radius a. (Fig. 5.1 in Podgoršak.)

Linear stopping power, dE/dx $=$ rate of energy loss per unit path length of charged particle

Mass stopping power, $S = -\rho^{-1} dE/dx$, is the commonly used measure of stopping power (in units $\mathrm{MeV\,m^2\,kg^{-1}}$)

2 types of stopping powers:

1. *Radiative stopping power*, S_{rad} – for radiative collisions; only light charged particles (i.e. electrons and positrons) experience appreciable energy losses; can result in *bremsstrahlung emission*
2. *Collision stopping power*, S_{col} – for hard and soft collisions involving both light and heavy charged particles; can result in *atomic excitation and ionisation*

$$S_{\mathrm{tot}} = S_{\mathrm{rad}} + S_{\mathrm{col}} \qquad \textit{total stopping power}$$

1.2.1 Radiative Stopping Power

For electrons and positrons:

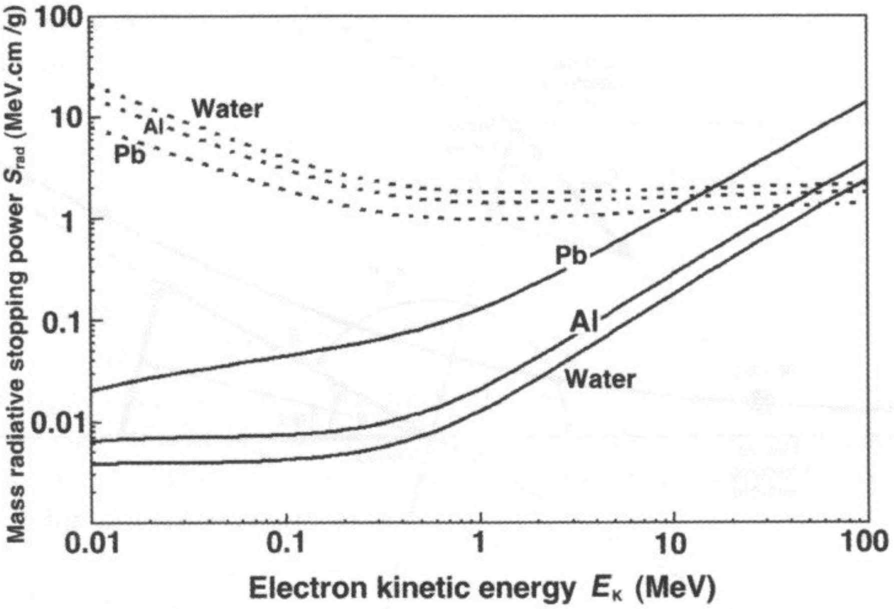
$$S_{\mathrm{rad}} = \frac{N_{\mathrm{A}}}{A} \sigma_{\mathrm{rad}} E_{\mathrm{i}} \tag{9}$$

$E_{\mathrm{i}} = E_{\mathrm{K,i}} + m_{\mathrm{e}} c^2$ $=$ initial total energy $E_{\mathrm{K,i}}$ $=$ initial kinetic energy σ_{rad} $=$ total cross section for bremmstrahlung production

S_{rad} can be written in terms of a weakly varying function B_{rad} of Z and E_{i} (see Table 5.1 in in Podgoršak):

$$S_{\mathrm{rad}} = \alpha r_{\mathrm{e}}^2 Z^2 \frac{N_{\mathrm{A}}}{A} B_{\mathrm{rad}} E_{\mathrm{i}} \tag{10}$$

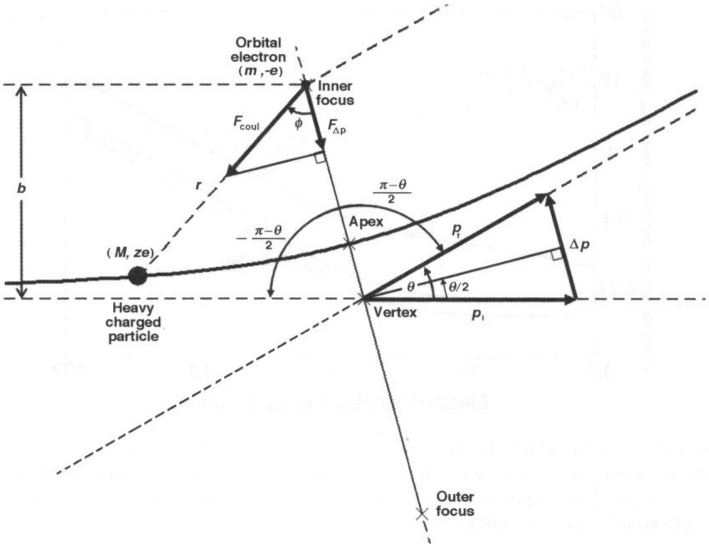
derived theoretically by Bethe and Heitler.



Radiative stopping powers for electrons in different material (solid curves) and collision stopping powers (dashed curves) for the same material. (Fig. 5.2 in Podgoršak.)

1.2.2 Collision Stopping Power (Heavy Particles)

- for $E_\mathrm{i} \lesssim 10\,\mathrm{MeV}$, heavy charged particles undergo soft and hard collisions
- small angle scattering ($\theta \simeq 0$)



Schematic diagram of a heavy charged particle collision with an orbital electron. The scattering angle θ is exaggerated for clarity. (Fig. 5.3 in Podgoršak.)

Classical Derivation

Momentum transfer:

$$\Delta p = \int F_{\Delta p} dt = \int_{-\infty}^{\infty} F_{\text{coul}} \cos\phi \, dt$$

where $F_{\text{coul}} = (ze^2/4\pi\varepsilon_0)r^{-2}$, giving

$$\Delta p = \frac{ze^2}{4\pi\varepsilon_0} \int_{-(\pi-\theta)/2}^{+(\pi-\theta)/2} \frac{\cos\phi}{r^2} \frac{dt}{d\phi} \, d\phi$$

Hyperbolic particle trajectory ⇒ angular displacement varies with time ⇒ $d\phi/dt = \omega$ and conservation of angular momentum requires $L = Mv_\infty b = M\omega r^2 \Rightarrow$

$$\begin{aligned} \Delta p &= \frac{ze^2}{4\pi\varepsilon_0}\frac{1}{v_\infty b} \int_{-(\pi-\theta)/2}^{+(\pi-\theta)/2} \cos\phi \, d\phi \\ &= 2\frac{ze^2}{4\pi\varepsilon_0}\frac{1}{v_\infty b} \cos\frac{\theta}{2} \\ &\approx 2\frac{ze^2}{4\pi\varepsilon_0}\frac{1}{v_\infty b} \qquad (11) \end{aligned}$$

Energy transferred to electron in a single collision with impact parameter b:

$$\Delta E(b) = \frac{(\Delta p)^2}{2m_{\text{e}}} = 2\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{z^2}{m_{\text{e}} v_\infty^2 b^2} \qquad (12)$$

Total energy loss obtained by integrating $\Delta E(b)$ over all possible b and accounting for all electrons available for interactions.

no. electrons in volume annulus between b and $b + db$

$$= \text{no. electrons per unit mass} \times \text{mass in annulus}$$

$$\Rightarrow \Delta n = \left(\frac{ZN_{\text{A}}}{A}\right) dm$$

where

$$dm = \rho \, dV = \rho[\pi(b+db)^2\Delta x - \pi b^2 \Delta x] \approx 2\pi \, \rho \, b \, db \, \Delta x$$

$$\Rightarrow \Delta n \approx 2\pi \, \rho \, (ZN_{\text{A}}/A) \, b \, db \, \Delta x$$

Multiply $\Delta E(b)$ by this and integrate over b to get the total energy transfer to electrons.

Mass collision stopping power

$$
\begin{aligned}
S_{\text{col}} = -\frac{1}{\rho}\frac{dE}{dx} &= 4\pi\,\frac{ZN_{\text{A}}}{A}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\frac{z^2}{m_{\text{e}}v_\infty^2}\int_{b_{\min}}^{b_{\max}}\frac{db}{b} \\
&= 4\pi\,\frac{ZN_{\text{A}}}{A}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\frac{z^2}{m_{\text{e}}v_\infty^2}\ln\frac{b_{\max}}{b_{\min}} \qquad (13)
\end{aligned}
$$

- $S_{\text{col}} \propto z^2$, where $z =$ atomic number of heavy charged particle (e.g. $z = 4$ for an α particle)

- $S_{\text{col}} \propto v_\infty^{-2}$, where $v_\infty =$ initial velocity of heavy charged particle

- $b_{\max} \Leftrightarrow \Delta E_{\min} =$ minimum energy transfer corresponding to minimum excitation or ionisation potential of orbital electron from (12)

$$
\Delta E_{\min} = 2\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\frac{z^2}{m_{\text{e}}v_\infty^2 b_{\max}^2} = I \qquad (14)
$$

$I =$ *mean ionisation-excitation potential* of medium

$$
I \approx 9.1Z(1+1.9Z^{-2/3})\,\text{eV} \qquad (15)
$$

e.g. $I \approx 78\,\text{eV}$ for carbon. But (15) is poor approximation for compounds (e.g. $I \approx 75\,\text{eV}$ for water).

- $b_{\min} \Leftrightarrow \Delta E_{\max} =$ maximum energy transfer corresponding to head-on collisions: $\Delta E_{\max} \approx 4\frac{m_{\text{e}}}{M}E_{\text{K,i}} = 2m_{\text{e}}v_\infty^2$ (for $M \gg m_{\text{e}}$), so

$$
\Delta E_{\max} = 2\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\frac{z^2}{m_{\text{e}}v_\infty^2 b_{\min}^2} = 2m_{\text{e}}v_\infty^2 \qquad (16)
$$

Putting together (14) and (16) gives

$$
\frac{b_{\max}}{b_{\min}} = \left(\frac{\Delta E_{\max}}{\Delta E_{\min}}\right)^{1/2} = \left(\frac{2m_{\text{e}}v_\infty^2}{I}\right)^{1/2} \qquad (17)
$$

$\Longrightarrow$ *classical collision stopping power* for heavy charged particles:

$$
S_{\text{col}} = 4\pi\,\frac{ZN_{\text{A}}}{A}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\frac{z^2}{m_{\text{e}}v_\infty^2}\,\frac{1}{2}\ln\frac{2m_{\text{e}}v_\infty^2}{I} \qquad (18)
$$

Generalised solution for the collision stopping power for heavy charged particles:

$$\begin{aligned} S_{\mathrm{col}} &= 4\pi \, \frac{N_{\mathrm{A}}}{A} \left(\frac{e^2}{4\pi\varepsilon_0} \right)^2 \frac{z^2}{m_{\mathrm{e}} c^2 (v_\infty/c)^2} \, B_{\mathrm{col}} \\ &\approx 3.070 \times 10^{-5} \frac{z^2}{A\beta^2} \, B_{\mathrm{col}} \ \mathrm{MeV\,m^2\,kg^{-1}} \end{aligned} \tag{19}$$

with A in units of kg and where $\beta = v_\infty/c$ and $B_{\mathrm{col}} =$ *atomic stopping number* includes relativistic and quantum-mechanical corrections and is $\propto Z$

	B_{col}
classical (Bohr)	$Z \ln \left(\frac{2m_{\mathrm{e}}v^2}{I} \right)^{1/2}$
non-rel, qm (Bethe-Bloch)	$Z \ln \left(\frac{2m_{\mathrm{e}}v^2}{I} \right)$
rel, qm (Bethe)	$Z \left[\ln \left(\frac{2m_{\mathrm{e}}c^2}{I} \right) + \ln \left(\frac{\beta^2}{1-\beta^2} \right) - \beta^2 \right]$
rel, qm, shell, polarisation	$Z \left[\ln \left(\frac{2m_{\mathrm{e}}c^2}{I} \right) + \ln \left(\frac{\beta^2}{1-\beta^2} \right) - \beta^2 - \frac{C_K}{Z} - \delta \right]$

- $C_K/Z =$ correction accounting for non-participation of K-shell electrons; important for low-$E_{\mathrm{K,i}}$

- $\delta =$ polarisation (density effect) correction; accounts for reduced participation by distant atoms resulting from effective Coulomb field being reduced by dipole of nearby atoms; important for light charged particles

Example: The stopping power of water for protons.
Using the Bethe formula (relativistic and quantum-mechanical derivation, but without shell and polarisation corrections), with $z = 1$ for protons and for H_2O, $A = 18.0\,\mathrm{g} = 0.0180\,\mathrm{kg}$, $Z = 10$, and $I = 75\,\mathrm{eV}$ giving

$$S_{\mathrm{col}} = 1.71 \times 10^{-2} \beta^{-2} \left[9.520 + \ln \left(\frac{\beta^2}{1-\beta^2} \right) - \beta^2 \right]$$

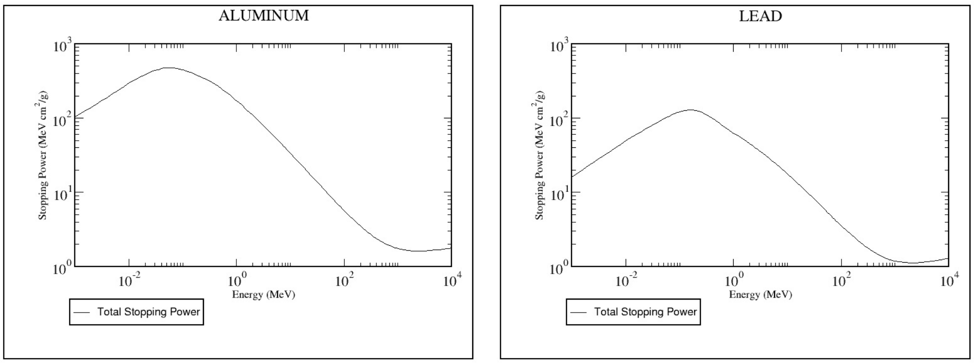
in units of $\mathrm{MeV\,m^2\,kg^{-1}}$. For $1\,\mathrm{MeV}$ protons, for instance, $\beta^2 = 0.00213$, giving

$$S_{\mathrm{col}} = 26.97\,\mathrm{MeV\,m^2\,kg^{-1}}$$

which compares well with the exact value obtained from the NIST/pstar database: $S_{\mathrm{col}} = 26.06\,\mathrm{MeV\,m^2\,kg^{-1}}$.

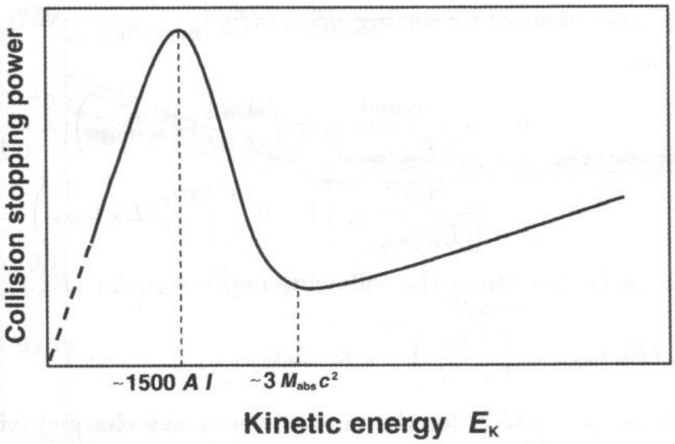
- $S_{\mathrm{col}} \propto Z/A$, but Z/A does not vary appreciably between different materials ($Z/A \approx 0.4 - 0.5$ typically)

- Z dependence of S_{col} mostly through I, which increases with Z; B_{col} has term $-\ln I$, so stopping power decreases with higher Z



Stopping powers of protons in aluminium (Z=13) and lead (Z=82) (data from NIST/pstar).

- dependence of S_{col} on particle kinetic energy E_{K} varies from non-relativistic to relativistic regimes



Schematic plot of the mass collision stopping power for a heavy charged particle as a function of kinetic energy. (Fig. 5.4 in Podgoršak.)

1.2.3 Collision Stopping Power (Light Particles)

3 differences from heavy particle collisions:

1. relativistic effects important at lower energies

2. larger fractional energy losses

3. radiative losses can occur

Hard and soft collisions combined using Møller and Bhabba cross sections for electrons and positrons, respectively.

$$S_{\rm col} = 2\pi r_{\rm e}^2 \frac{Z N_{\rm A}}{A} \frac{m_{\rm e}c^2}{\beta^2} \left[\ln\left(\frac{E_{\rm K}(1+\tau/2)}{I} \right) + F^{\pm}(\tau) - \delta \right] \tag{20}$$

where

$$F^{-}(\tau) = (1-\beta^2)[1+\tau^2/8-(2\tau+1)\ln 2] \text{ for electrons}$$

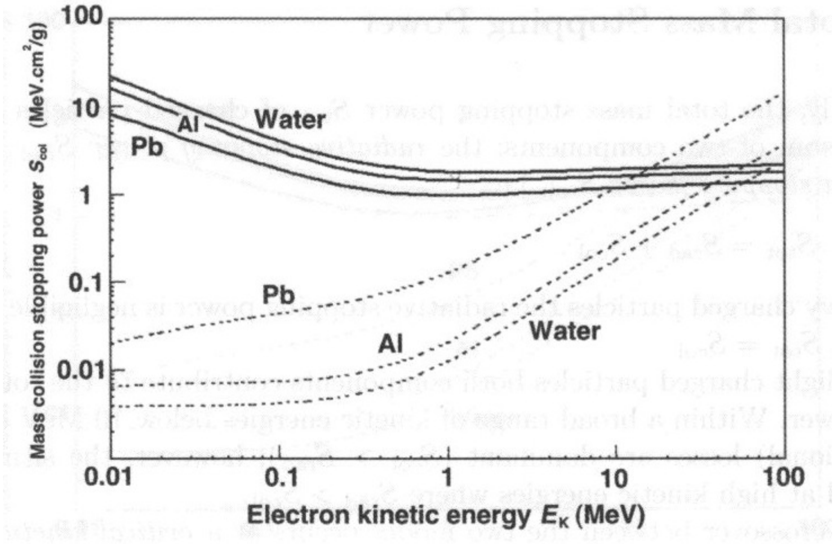
and

$$F^{+}(\tau) = 2\ln 2 - \frac{\beta^2}{12}\left[23 + \frac{14}{\tau+2} + \frac{10}{(\tau+2)^2} + \frac{4}{(\tau+2)^3} \right]$$

for positrons and where

$$\tau = \frac{E_{\rm K}}{m_{\rm e}c^2}$$

For light charged particles, $S_{\rm col}$ dependence on Z is similar to that for heavy charged particles, but dependence on $E_{\rm K}$ differs:



Mass collision stopping power (solid curves) and radiative stopping power (dashed curves) for electrons. (Fig. 5.5 in Podgoršak.)

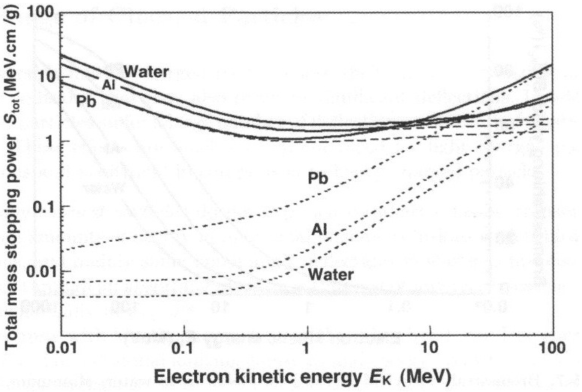
1.2.4 Total Mass Stopping Power

$$S_{\rm tot} = S_{\rm rad} + S_{\rm col} \tag{21}$$

- for heavy charged particles, $S_{\rm rad} \approx 0$
- for light charged particles, $S_{\rm col} > S_{\rm rad}$ for $E_{\rm K} \lesssim 10\,{\rm MeV}$ typically

- *critical kinetic energy*, $(E_\mathrm{K})_\mathrm{crit}$, where $E_\mathrm{col} = E_\mathrm{rad}$

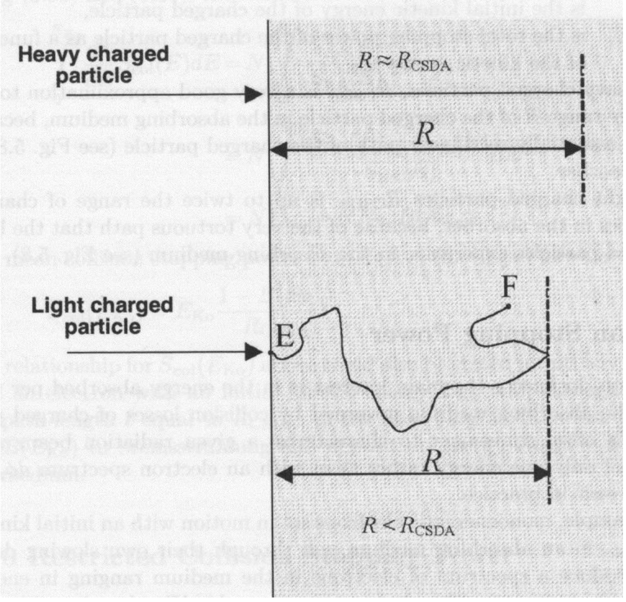
$$(E_\mathrm{K})_\mathrm{crit} \approx \frac{800\,\mathrm{MeV}}{Z} \tag{22}$$



Total mass stopping power (solid curves) and radiative and collision stopping power (dashed curves) for electrons. (Fig. 5.6 in Podgoršak.)

1.3 Range

- heavy charged particles experience small fractional energy losses and small angle deflections in elastic collisions
- light charged particles experience larger fractional energy losses and large angle deflections per elastic or inelastic collision



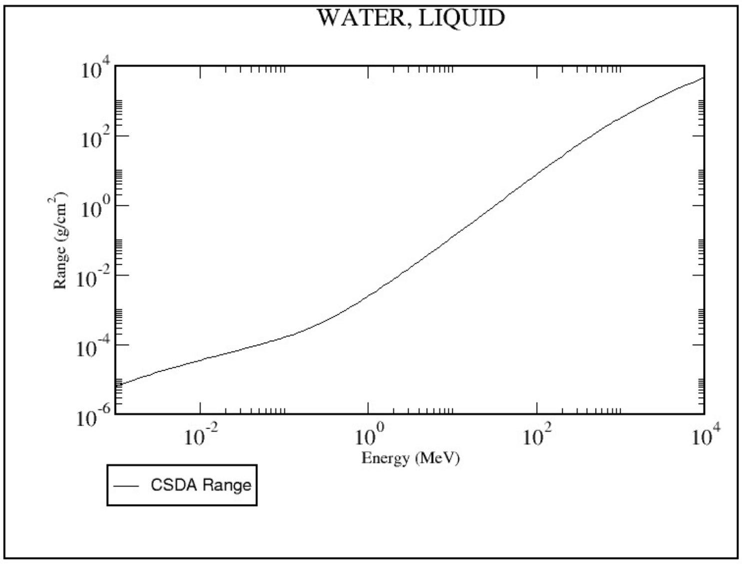
- *range*, R, of a particular charged particle in a particular medium measures the expected linear distance the particle will reach in that medium before coming to rest (i.e. cannot penetrate beyond R)
- depends on particle charge and kinetic energy, as well as absorber composition
- *CSDA range*, R_{CSDA}, measures average path length traversed by charged particles of a specific type in a given medium (in units $\mathrm{kg\,m^{-2}}$) in the *continuous slowing down approximation*
- $R_{\mathrm{CSDA}} > R$ always

$$R_{\mathrm{CSDA}} = \int_0^{E_{\mathrm{K,i}}} \frac{dE_{\mathrm{K}}}{S_{\mathrm{tot}}(E_{\mathrm{K}})} = -\rho \int_0^{E_{\mathrm{K,i}}} \frac{dE_{\mathrm{K}}}{dE_{\mathrm{K}}/dx} \tag{23}$$

- R_{CSDA} difficult to solve using analytic $S_{\mathrm{tot}}(E_{\mathrm{K}})$ solutions, (19) and (20)
- for light particles, need to also take into account radiative losses
- for heavy particles, $S_{\mathrm{tot}}(E_{\mathrm{K}}) = S_{\mathrm{col}}(E_{\mathrm{K}}) \propto z^2 B_{\mathrm{col}}(\beta)/\beta^2$, where β is related to E_{K} via $E_{\mathrm{K}} = (\gamma - 1)Mc^2$, where $\gamma = (1-\beta^2)^{-1/2}$, so $E_{\mathrm{K}} = E_{\mathrm{K}}(\beta)$ and
$$R_{\mathrm{CSDA}} \propto \int \frac{\beta^2\, dE_{\mathrm{K}}(\beta)}{z^2 B_{\mathrm{col}}(\beta)}$$
- use $dE_{\mathrm{K}} = Mg(\beta)d\beta$ and let $G(\beta) = B_{\mathrm{col}}(\beta)/\beta^2$:
$$R_{\mathrm{CSDA}} \propto \frac{M}{z^2} \int_0^{\beta} \frac{g(\beta)}{G(\beta)}\, d\beta = \frac{M}{z^2} f(\beta)$$
- $f(\beta)$ independent of heavy particle type (only depends on β) ⇒ can calculate values of R_{CSDA} for heavy particles relative to protons

$$R_{\mathrm{CSDA}}(\beta) = \frac{M}{M^{\mathrm{p}} z^2} R^{\mathrm{p}}_{\mathrm{CSDA}}(\beta) \tag{24}$$

$R^{\mathrm{p}}_{\mathrm{CSDA}}(\beta)$ = proton range M/M^{p} = heavy charged particle mass / proton mass
z = atomic number of heavy charged particle



Range of protons in water ($\rho = 1\,\mathrm{g\,cm^{-3}}$ so depth in cm has same value as R). From the NIST/pstar database.

Example 1: Range of an $80\,\mathrm{MeV}$ ${}^3He^{2+}$ ion in soft tissue.
We have $z = 2$ and $M = 3M^{\mathrm{p}}$, so $R(\beta) = \frac{3}{4}R^{\mathrm{p}}(\beta)$. Now we need to find the energy of a proton having the same β as the ${}^3He^{2+}$ ion. For a fixed β, $E_{\mathrm{K}}/M = const$, so $E_{\mathrm{K}}^{\mathrm{p}} = (M^{\mathrm{p}}/M)E = 80/3\,\mathrm{MeV} = 26.7\,\mathrm{MeV}$. Using the NIST/pstar database, and using water as a soft tissue equivalent,

$$R_{\mathrm{CSDA}}^{\mathrm{p}} = 0.7173\,\mathrm{g\,cm^{-2}} = 7.173\,\mathrm{kg\,m^{-2}}$$

$$\Longrightarrow R_{\mathrm{CSDA}} = 0.5380\,\mathrm{g\,cm^{-2}} = 5.380\,\mathrm{kg\,m^{-2}}$$

Since water has $\rho = 1\,\mathrm{g\,cm^{-3}}$, the average distance a ${}^3He^{2+}$ ion can penetrate into soft tissue is $\approx 0.5\,\mathrm{cm}$. Note: this exceeds the minimum thickness of outer layer of dead skin cells (epidermis, $\sim 0.007\,\mathrm{cm}$), so ${}^3He^{2+}$ ions can reach living cells from outside the human body.

Example 2: Range of a $7.69\,\mathrm{MeV}$ α particle in soft tissue.
Using $z = 2$ and $M = 4M^{\mathrm{p}}$ gives $R^{\alpha}(\beta) = R^{\mathrm{p}}(\beta)$. For the same β, the proton energy is $E_{\mathrm{K}}^{\mathrm{p}} = (7.69/4)\,\mathrm{MeV} \approx 1.923\,\mathrm{MeV}$. For this proton energy, the NIST/pstar database gives $R^{\mathrm{p}} = 7.077 \times 10^{-3}\,\mathrm{g\,cm^{-2}}$. So the average depth to which $7.69\,\mathrm{MeV}$ α particles can penetrate into soft tissue is close to the thickness of the epidermis. This means that external sources of these particles are less of a health hazard than ${}^{3}He^{2+}$ ions. However, $7.69\,\mathrm{MeV}$ α particles are emitted by the radon daughter ${}^{214}_{84}Po$, which is present in the atmosphere of uranium mines. These α's pose a serious radiological hazard when ingested through the lungs. This has been linked to the higher incidence of lung cancer among uranium miners.

1.4 Mean Stopping Power

- in practice, charged particle beams are generally not monoenergetic
- electrons in an initially monoenergetic beam will lose different amounts of energy through a medium
- produces an energy *spectrum*:

$$\frac{d\phi(E)}{dE} = \frac{N}{S_{\mathrm{tot}}(E)} \tag{25}$$

- $N =$ no. of monoenergetic electrons of initial kinetic energy $E_{\mathrm{K,0}}$ per unit mass in medium
- collision stopping power for a single energy $E_{\mathrm{K,0}}$ should be defined as an average over energy spectrum produced as a result of all collisions:

$$\overline{S}_{\mathrm{col}}(E_{\mathrm{K,0}}) = \frac{\int_0^{E_{\mathrm{K,0}}} \frac{d\phi}{dE} S_{\mathrm{col}}(E)\,dE}{\int_0^{E_{\mathrm{K,0}}} \frac{d\phi}{dE}\,dE} \tag{26}$$

Using the definition for R_{CSDA}:

$$\int_0^{E_{\mathrm{K,0}}} \frac{d\phi}{dE}\,dE = N \int_0^{E_{\mathrm{K,0}}} \frac{dE}{S_{\mathrm{tot}}(E)} = N R_{\mathrm{CSDA}}$$

Similarly,

$$\int_0^{E_{\mathrm{K,0}}} \frac{d\phi}{dE} S_{\mathrm{col}}(E)\,dE = N \int_0^{E_{\mathrm{K,0}}} \frac{S_{\mathrm{col}}(E)}{S_{\mathrm{tot}}(E)}\,dE$$

and $S_{\text{col}} = S_{\text{tot}} - S_{\text{rad}}$ implies

$$\begin{aligned} \int_0^{E_{\text{K},0}} \frac{d\phi}{dE} S_{\text{col}}(E)\, dE &= N \int_0^{E_{\text{K},0}} \left[1 - \frac{S_{\text{rad}}(E)}{S_{\text{tot}}(E)} \right] dE \\ &= N E_{\text{K},0} \left[1 - B(E_{\text{K},0}) \right] \end{aligned} \tag{27}$$

where

$$B(E_{\text{K},0}) = \frac{1}{E_{\text{K},0}} \int_0^{E_{\text{K},0}} \frac{S_{\text{rad}}(E)}{S_{\text{tot}}(E)}\, dE \qquad \textit{radiation yield} \tag{28}$$

Putting together gives the *mean collision stopping power*:

$$\overline{S}_{\text{col}}(E_{\text{K},0}) = E_{\text{K},0}\, \frac{1 - B(E_{\text{K},0})}{R_{\text{CSDA}}} \tag{29}$$

For heavy charged particles, $B(E_{\text{K},0}) = 0$, so $\overline{S}_{\text{col}}(E_{\text{K},0}) = E_{\text{K},0}/R_{\text{CSDA}}$.

Example: Mean stopping power for a $5\,\text{MeV}$ α in lead.
Since $B(E_{\text{K},0}) = 0$, then $\overline{S}_{\text{col}}(E_{\text{K},0}) = E_{\text{K},0}/R_{\text{CSDA}}$. From the NIST/astar database, we find $R_{\text{CSDA}} = 1.702 \times 10^{-2}\text{g}\,\text{cm}^{-2}$, so

$$\overline{S}_{\text{col}}(E_{\text{K},0}) = 2.94 \times 10^{2}\,\text{MeV}\,\text{cm}^{2}\,\text{g}^{-1} = 29.4\,\text{MeV}\,\text{m}^{2}\,\text{kg}^{-1}$$

c.f. the collision stopping power is $S_{\text{col}} = 23.3\,\text{MeV}\,\text{m}^{2}\,\text{kg}^{-1}$.