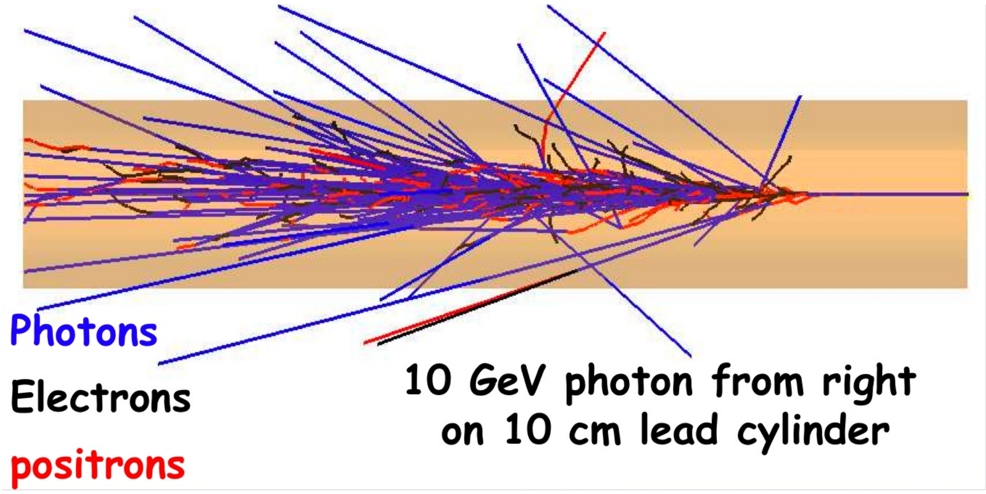
PHYS 5012
Radiation Physics and Dosimetry

Lecture 2

Tuesday 10 March 2009

Contents

1 Interactions of Photons with Matter



Simulation of a photon penetrating a slab of lead, showing the secondary particles produced as a result of the photon's interactions with Pb atoms.

1.1 General Aspects

- photon interactions in matter involve either *nuclei* or *orbital electrons* of an atom

nucleus	orbital electrons
nuclear	Thomson scattering
photodisintegration	Compton scattering
electromagnetic	triplet production
pair production	photoelectric effect

- orbital electron–photon interactions occur with *loosely bound or free electrons* (binding energy $E_{\mathrm{B}} \ll h\nu$) or *tightly bound electrons* ($E_{\mathrm{B}} \lesssim h\nu$)

- photon interactions with tightly bound electrons are considered interactions with whole atom

Photons lose energy in very few large interactions, resulting in:

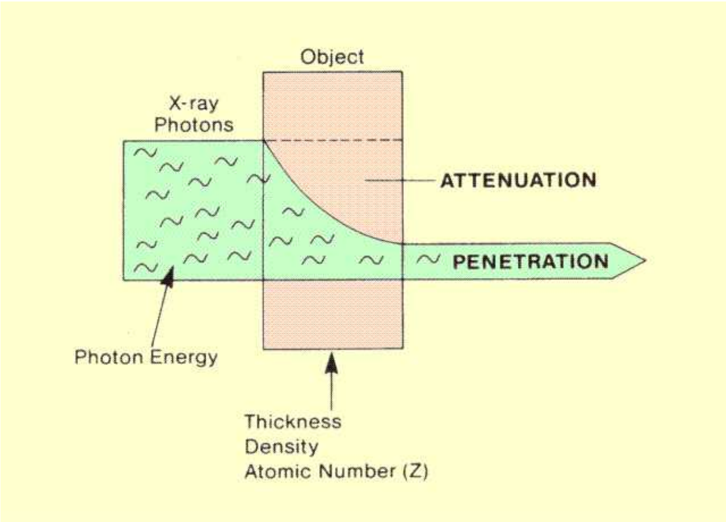
1. *photon absorption* and energy transferred to light charged particles (electrons, positrons)

2. *photon scattering* – elastic (coherent), with no energy transfer to electrons, or inelastic (incoherent), with energy transfer to electrons

Light charged particles produced as a result of photon interactions with atoms deposit their kinetic energy to the medium via many small collisions, resulting in *ionisation losses* or *radiative losses*.

1.2 Attenuation Coefficients

- *linear attenuation coefficient*, μ (units m^{-1}), measures probability per unit path length of a photon interaction in an absorber; depends on $h\nu$ and Z



$\mu\Delta x \gg 1 \Longrightarrow$ strong attenuation of primary photon beam

Example: water

- for $100\,\mathrm{keV}$ photons, $\mu = 17.1\,\mathrm{m}^{-1} = 0.171\,\mathrm{cm}^{-1} \Longrightarrow$ 17.1% of photons interact per cm of water (on average)
- essentially all $100\,\mathrm{keV}$ photons will have an interaction in just $(0.171\,\mathrm{cm}^{-1})^{-1} \approx 5.8\,\mathrm{cm}$ of water
- for $2\,\mathrm{MeV}$, $\mu = 4.94\,\mathrm{m}^{-1} = 0.0494\,\mathrm{cm}^{-1} \Longrightarrow$ 4.9% of photons interact per cm of water (on average)
- essentially all $2\,\mathrm{MeV}$ photons will have an interaction in $20.2\,\mathrm{cm}$ of water

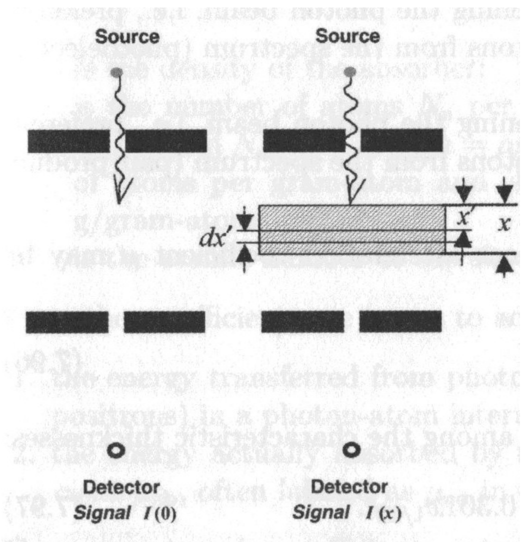
Example: lead

- for $2\,\mathrm{MeV}$ photons, $\mu = 0.514\,\mathrm{cm}^{-1} \Longrightarrow$ 51% of photons interact per cm of lead (on average)
- essentially all $2\,\mathrm{MeV}$ photons will have an interaction in just $1.9\,\mathrm{cm}$ of lead

1.2.1 Narrow Beam Geometry

- μ measured experimentally for different absorbers using a thin slab of thickness x placed between a narrowly collimated source of monoenergetic photons and a narrowly collimated detector

- intensity signal I measured by detector before and after slab introduced



Measurement of linear attenuation coefficient using narrow beam geometry. Incident photon beam from source must be sufficiently collimated to prevent scattered photons from hitting detector, so only attenuated primary beam is measured. Source should be sufficiently far away that photon beam is normally incident on the slab. (Fig. 7.31a in Podgoršak)

An incremental layer dx' in the slab reduces beam intensity I according to

$$dI = -\mu I \, dx'$$

and integration over the whole slab, $0 \leq x' \leq x$, gives

$$I(x) = I(0) \exp\left(-\int_0^x \mu \, dx'\right) \qquad \textit{exponential attenuation} \qquad (1)$$

where $I(0)$ = intensity of primary (incident) photon beam.

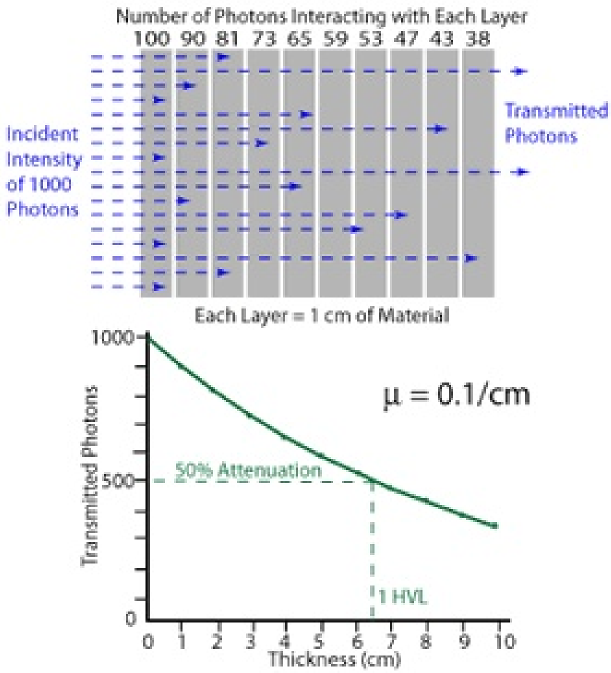
- for a homogeneous medium, $\mu = const.$, so $I(x) = I(0)\exp(-\mu x)$

- since $I \propto N$, where $N =$ no. of photons, the exponential attenuation law extends to N, i.e.
$$N = N_0 \exp(-\int_0^x \mu \, dx')$$

- *no. of interactions* = $\Delta N = N_0 - N = N_0(1 - e^{-\mu x})$

- $\Delta N \neq dN = -\mu N \, dx$ unless x is very small

Example: $N_0 = 1000$ photons incident on $x = 10\,\mathrm{cm}$ of absorbing material with $\mu = 0.1\,\mathrm{cm}^{-1}$.
$N = 1000(1 - e^{-1}) = 368 \Longrightarrow \Delta N = 632$ interactions.



1.2.2 Characteristic Thicknesses

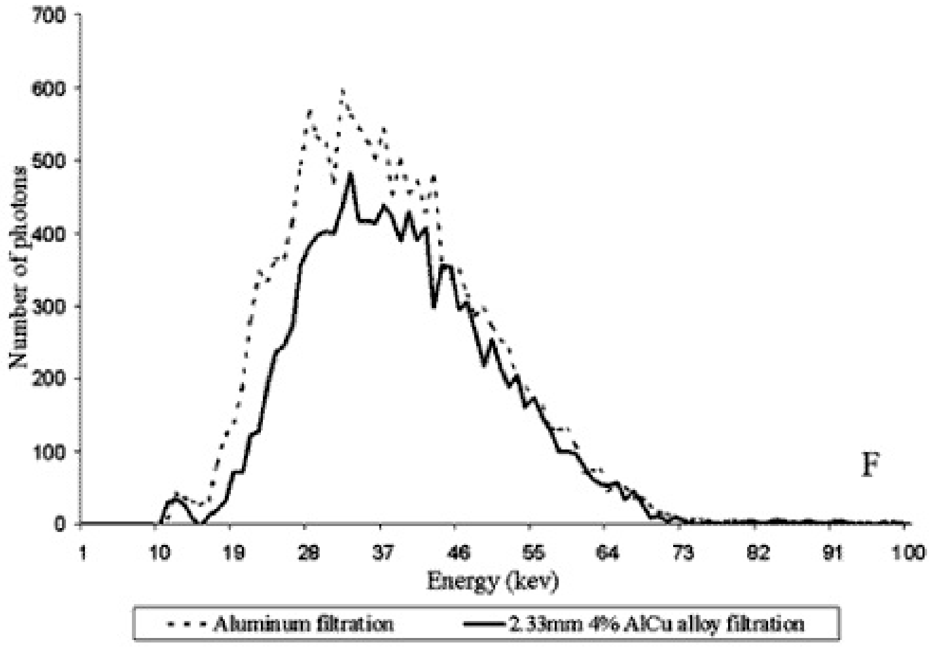
Some commonly used definitions for monoenergetic photons in narrow beam geometry:

- *Mean Free Path*, MFP $= \bar{x} = \mu^{-1}$ average distance a photon travels before undergoing an interaction; $I(\bar{x})/I(0) = e^{-1} = 0.368$
- *Half Value Layer*, HVL $= x_{1/2}$ thickness at which 50% of the primary beam is attenuated: $I(x_{1/2})/I(0) = 0.5 \Longrightarrow x_{1/2} = \ln 2/\mu$
- *Tenth Value Layer*, TVL $= x_{1/10}$; thickness at which beam is attenuated to 10% incident intensity: $I(x_{1/10})/I(0) = 0.1 \Longrightarrow x_{1/10} = \ln 10/\mu$
- *spectral effects* can be important in beam attenuation
- higher energy photons generally penetrate deeper (μ generally decreases with E)

- *second half value layer,* HVL_2 is thickness where I is attenuated from $0.5I(0)$ to $0.25I(0)$, i.e. $x_{1/4} - x_{1/2}$ and

$$\chi = \frac{x_{1/2}}{x_{1/4} - x_{1/2}} = \frac{HVL}{HVL_2}$$ *homogeneity factor*

- $\chi \neq 1 \Longrightarrow$ photon beam has spectral energy distribution
- $\chi < 1 \Longrightarrow$ *spectral hardening* (absorber preferentially removes low-E photons)
- $\chi > 1 \Longrightarrow$ *spectral softening* (absorber preferentially removes high-E photons)



A bremsstrahlung spectrum produced by a thick target in an X-ray unit is often filtered with aluminium, which hardens the spectrum. Filtration with an AlCu alloy produces a spectrum that is harder.

1.2.3 Other Coefficients

Summary of other photon attenuation coefficients and their relation to μ:

	symbol	definition	units
linear attenuation coefficient	μ	μ	m^{-1}
mass attenuation coefficient	μ_m	μ/ρ	$\mathrm{m}^2\,\mathrm{kg}^{-1}$
atomic cross-section	${}_\mathrm{a}\mu$	$\mu/n^\sqcap$	$\mathrm{m}^2\,\mathrm{atom}^{-1}$
electronic cross-section	${}_\mathrm{e}\mu$	$\mu/(Zn^\sqcap)$	$\mathrm{m}^2\,\mathrm{electron}^{-1}$

- $\rho =$ density of absorber
- $n^{\sqcap} =$ no. atoms per unit volume $= \rho N_{\text{A}}/A$, where $N_{\text{A}} = 6.022 \times 10^{23}$atoms is Avagadro's number and $A =$ atomic mass of absorber
- N_{A} is no. of atoms in a mass (in grams) equivalent to A; e.g. for carbon, $A = 12$, so $12\,\text{g}$ of C contains N_{A} atoms
- *mass attenuation coefficient*, μ_{m}, is just a mass-weighted cross-section (area per unit mass)
- the attenuation factor is $\mu_{\text{m}}\rho x$ instead of μx

Example: The attenuation of a $1\,\text{mm}$ thick copper plate, given $\mu_{\text{m}} = 0.420\,\text{cm}^2\,\text{g}^{-1}$ and $\rho = 8.96\,\text{g}\,\text{cm}^{-3}$ for Cu, is

$$\frac{N}{N_0} = \exp(-\mu_{\text{m}}\rho x) = \exp\left(-0.420 \cdot 8.96 \cdot 0.1\right) = 0.69$$

- if ρ is unknown, effective "thickness", ρx, of absorber can be deduced from weight and area; e.g. a $10 \times 10\,\text{cm}^2$ piece of foil weighs $0.03\,\text{kg}$, so its thickness is $\rho x = 0.03\,\text{kg}/(0.1\,\text{m})^2 = 3\,\text{kg}\,\text{m}^{-2}$

Two other commonly used coefficients are:

1. *energy transfer coefficient*

$$\mu_{\text{tr}} = \mu\frac{\bar{E}_{\text{tr}}}{h\nu} \tag{2}$$

   where $\bar{E}_{\text{tr}} =$ average photon energy transferred to KE of light charged particles

2. *energy absorption coefficient*

$$\mu_{\text{ab}} = \mu\frac{\bar{E}_{\text{ab}}}{h\nu} = \mu\frac{\bar{E}_{\text{tr}} - \bar{E}_{\text{rad}}}{h\nu} \tag{3}$$

   where $\bar{E}_{\text{ab}} =$ average energy absorbed in a volume of interest in the medium and $\bar{E}_{\text{rad}} =$ average radiative energy lost by charged particles

The energy absorption coefficient can be written as

$$\mu_{\text{ab}} = \mu_{\text{tr}}\left(1 - \frac{\bar{E}_{\text{rad}}}{\bar{E}_{\text{tr}}}\right) = \mu_{\text{tr}}(1 - \bar{g}) \tag{4}$$

where $\bar{g}$ is the *radiative fraction* – i.e. the average fraction of energy transferred by photon interactions that is subsequently lost to radiative interactions by secondary charged particles. This is mostly bremsstrahlung losses.

- for low-Z materials, and low-$h\nu$, $\bar{g} \to 0$ and $\mu_{\rm tr} \approx \mu_{\rm ab}$
- $\bar{g}$ increases with Z or $h\nu$ – e.g. for lead ($Z = 82$), with $h\nu = 10\,\mathrm{MeV}$, $\bar{g} = 0.26$ and $\mu_{\rm ab} = 0.74\mu_{\rm tr}$

Example: carbon ($Z = 6$)
For $10\,\mathrm{MeV}$ photons, $\mu_{\rm m} = \mu/\rho = 0.0196\,\mathrm{cm}^2\,\mathrm{g}^{-1}$ and a mass energy transfer coefficient $\mu_{\rm tr}/\rho = 0.0143\,\mathrm{cm}^2\,\mathrm{g}^{-1}$ and a mass energy absorption coefficient $\mu_{\rm ab}/\rho = 0.0138\,\mathrm{cm}^2\,\mathrm{g}^{-1}$. So (2) and (3) imply

$$\bar{E}_{\rm tr} = \frac{\mu_{\rm tr}/\rho}{\mu/\rho} h\nu \approx 7.30\,\mathrm{MeV}$$

$$\bar{E}_{\rm ab} = \frac{\mu_{\rm ab}/\rho}{\mu/\rho} h\nu \approx 7.04\,\mathrm{MeV}$$

so a substantial fraction of the incident photon energy is transferred to charged particles and virtually all the transferred energy is absorbed by the carbon atoms; a negligible amount is lost to radiation:

$$\bar{g} = 1 - \left(\frac{\mu_{\rm ab}/\rho}{\mu_{\rm tr}/\rho}\right) \approx 0.035$$

Example: copper ($Z = 29$)
For $10\,\mathrm{MeV}$ photons, $\mu_{\rm m} = \mu/\rho = 0.0310\,\mathrm{cm}^2\,\mathrm{g}^{-1}$ and a mass energy transfer coefficient $\mu_{\rm tr}/\rho = 0.0248\,\mathrm{cm}^2\,\mathrm{g}^{-1}$ and a mass energy absorption coefficient $\mu_{\rm ab}/\rho = 0.0215\,\mathrm{cm}^2\,\mathrm{g}^{-1}$. So (2) and (3) imply

$$\bar{E}_{\rm tr} = \frac{\mu_{\rm tr}/\rho}{\mu/\rho} h\nu \approx 8.0\,\mathrm{MeV}$$

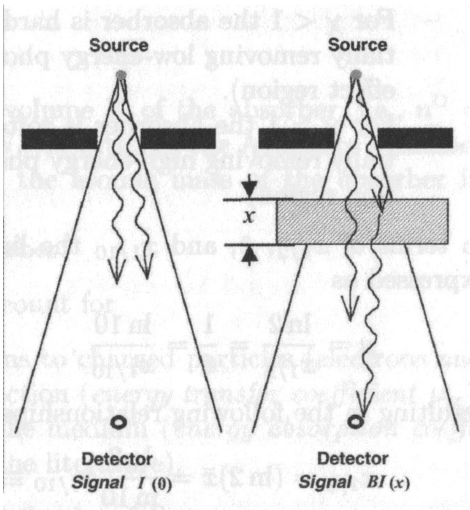
$$\bar{E}_{\rm ab} = \frac{\mu_{\rm ab}/\rho}{\mu/\rho} h\nu \approx 6.9\,\mathrm{MeV}$$

so a slightly larger amount of photon energy is transferred compared to the case for carbon, but a smaller fraction of the transferred energy is absorbed; i.e. a larger fraction is lost to radiation:

$$\bar{g} = 1 - \left(\frac{\mu_{\rm ab}/\rho}{\mu_{\rm tr}/\rho}\right) \approx 0.13$$

1.2.4 Broad Beam Geometry

- if ideal narrow-beam geometry not attained, beam divergence results in scattered and secondary photons reaching a detector
- *narrow-beam attenuation* – scattered & secondary photons not counted (can still measure μ)
- *broad-beam attenuation* – scattered & secondary photons counted; simple exponential attenuation no longer valid; *effective attenuation*, $\mu' < \mu$



Broad beam geometry. B is the *build-up factor* that accounts for scattered and secondary photons that reach the detector. Broad beam geometry is used in radiation protection for designing shielding.
(Fig. 7.31b in Podgoršak)

1.3 Physical Processes

The attenuation coefficients are determined from several different competing physical processes, the most important ones being the photoelectric effect, Compton scattering, pair production, as well as Thomson and Rayleigh scattering. Each of these processes dominates at different photon energies and the *total* attenuation coefficient at a specific energy is obtained from the sum of the individual coefficients for each process. This is determined from the corresponding cross sections for photon interactions.

1.3.1 Thomson Scattering

- scattering of low-energy photons ($h\nu \ll m_e c^2$) off free (or loosely bound) electrons
- negligible energy transfer to electrons $\Rightarrow$ *coherent* scattering

- interaction can be adequately described classically: photons are sinusoidal variations of electromagnetic (EM) waves, electrons treated as classical harmonic oscillators

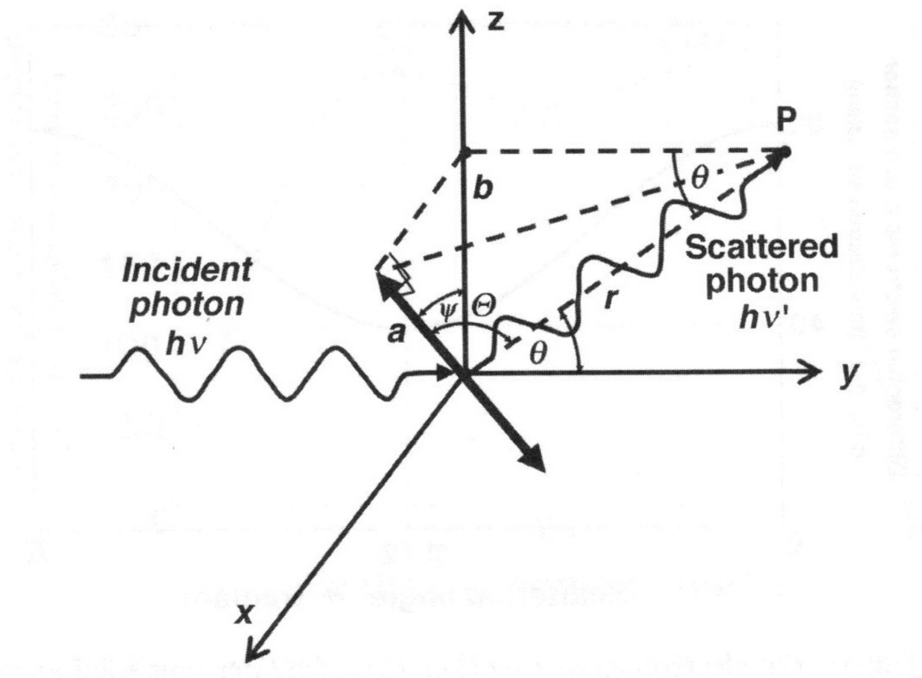
- EM waves re-emitted as a result of oscillating electric dipole

$$E_{\rm rad}^{\rm inc} = E_{\rm rad,0} \sin \omega t \qquad \text{incident EM field} \tag{5}$$

$$E_{\rm rad}^{\rm out} = \frac{e}{4\pi\varepsilon_0} \frac{\ddot{x} \sin \Theta}{rc^2} \qquad \text{emitted EM field} \tag{6}$$

- $E_{\rm rad,0}$ = amplitude of incident EM wave

- Θ = angle between emitted wave propagation direction $\hat{\mathbf{r}}$ and incident wave polarisation $\mathbf{E}_{\rm rad}^{\rm inc}$



- $\ddot{x}$ =electron acceleration, specified by equation of motion for harmonic oscillator:

$$m_{\rm e}\ddot{x} = eE_{\rm rad}^{\rm inc} = eE_{\rm rad,0} \sin \omega t$$

- use this to eliminate $\ddot{x}$ from (6), giving

$$E_{\rm rad}^{\rm out} = r_{\rm e} E_{\rm rad,0} \frac{\sin \omega t \sin \Theta}{r} \tag{7}$$

where $r_{\rm e} = e^2/(4\pi\varepsilon_0 m_{\rm e} c^2) \approx 2.818 \times 10^{-15}\,{\rm m}$ is the classical electron radius

- *electronic differential cross section* $d_{\rm e}\sigma_{\rm T}$ for re-emission into a solid angle $d\Omega$ is

$$d_{\rm e}\sigma_{\rm T} = \frac{\bar{S}^{\rm out}}{\bar{S}^{\rm inc}}\, dA = \frac{\bar{S}^{\rm out}}{\bar{S}^{\rm inc}} r^2 d\Omega$$

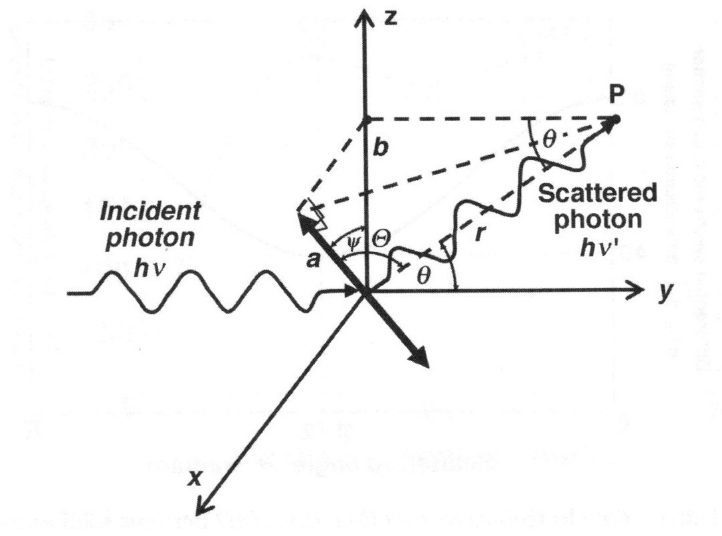
where $\bar{S} = c\varepsilon_0 \bar{E}_{\rm rad}^2 =$ average Poynting flux

$$\bar{S}^{\rm inc} = c\varepsilon_0 (\bar{E}_{\rm rad}^{\rm inc})^2 = c\varepsilon_0 E_{\rm rad,0}^2 \overline{\sin^2 \omega t} = \frac{1}{2} c\varepsilon_0 E_{\rm rad,0}^2 \tag{8}$$

$$\begin{aligned} \bar{S}^{\rm out} = c\varepsilon_0 (\bar{E}_{\rm rad}^{\rm out})^2 &= c\varepsilon_0 {r_{\rm e}}^2 E_{\rm rad,0}^2 \overline{\frac{\sin^2 \omega t \sin^2 \Theta}{r^2}} \\ &= \frac{1}{2} c\varepsilon_0 {r_{\rm e}}^2 E_{\rm rad,0}^2 \overline{\frac{\sin^2 \Theta}{r^2}} \end{aligned} \tag{9}$$

$$\Longrightarrow \frac{d_{\rm e}\sigma_{\rm T}}{d\Omega} = {r_{\rm e}}^2\, \overline{\sin^2 \Theta} \tag{10}$$

For unpolarised radiation, $\overline{\sin^2 \Theta}$ determined geometrically:



$$\cos\Theta = \frac{a}{r} \quad , \quad \sin\theta = \frac{b}{r} \quad , \quad \cos\psi = \frac{a}{b}$$

$$\Longrightarrow \cos\Theta = \sin\theta \cos\psi$$

- $\theta =$ *scattering angle* between incident and scattered photon
- $\psi =$ *polarisation angle*

$$\overline{\sin^2\Theta} = \overline{1-\cos^2\Theta} = \overline{1-\sin^2\theta\cos^2\psi} \tag{11}$$

Average over all polarisation angles:

$$\begin{aligned}\overline{\sin^2\Theta} &= \frac{\int_0^{2\pi} 1-\sin^2\theta\cos^2\psi\,d\psi}{\int_0^{2\pi} d\psi}\\ &= 1-\frac{1}{2\pi}\sin^2\theta\int_0^{2\pi}\cos^2\psi\,d\psi\\ &= 1-\frac{1}{2}\sin^2\theta = \frac{1}{2}(1+\cos^2\theta)\end{aligned} \tag{12}$$

$$\Longrightarrow \frac{d_{\mathrm{e}}\sigma_{\mathrm{T}}}{d\Omega} = \frac{1}{2}{r_{\mathrm{e}}}^2(1+\cos^2\theta) \text{ \textit{Thomson differential cross section}} \tag{13}$$

The *total* Thomson electronic cross section is obtained by integrating over solid angle:

$$\begin{aligned}{}_{\mathrm{e}}\sigma_{\mathrm{T}} &= \int \frac{d_{\mathrm{e}}\sigma_{\mathrm{T}}}{d\Omega}\,d\Omega\\ &= \frac{1}{2}{r_{\mathrm{e}}}^2\int_0^{2\pi} d\phi \int_0^{\pi}(1+\cos^2\theta)\,\sin\theta\,d\theta\\ &= \frac{8}{3}\pi{r_{\mathrm{e}}}^2 = 6.65\times10^{-29}\,\mathrm{m}^2\end{aligned} \tag{14}$$

Another commonly used unit for cross section is the "barn":

$$1\,\mathrm{b} = 10^{-24}\,\mathrm{cm}^2 = 10^{-28}\,\mathrm{m}^2$$

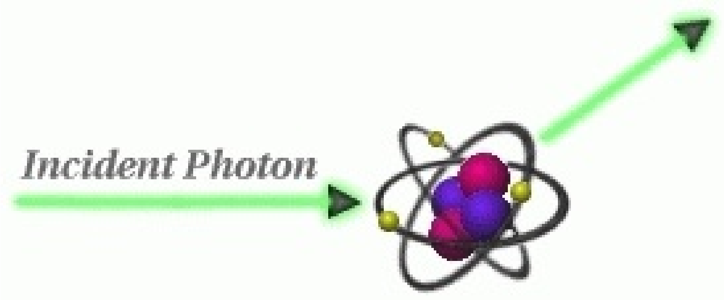
$$\Longrightarrow {}_{\mathrm{e}}\sigma_{\mathrm{T}} = 0.665\,\mathrm{b}$$

Note: the total Thomson cross section is *constant*; there is no explicit dependence on photon or electron energy. As $h\nu \to m_{\mathrm{e}}c^2$, however, classical theory breaks down.

1.3.2 Rayleigh Scattering

- photons scatter off bound atomic electrons
- coherent: total no. photons at each energy conserved

- negligible atomic recoil
- small angle scattering (small θ)
- can be important at low $h\nu$, for high Z materials

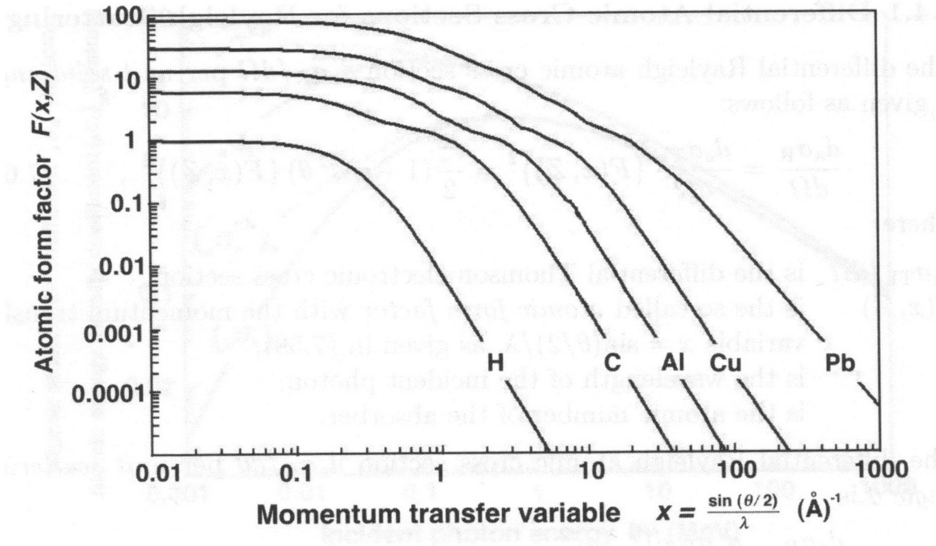


Differential atomic cross section:

$$\frac{d_{\rm a}\sigma_{\rm R}}{d\Omega} = \frac{d_{\rm e}\sigma_{\rm T}}{d\Omega}\,[F(x,Z)]^2 \tag{15}$$

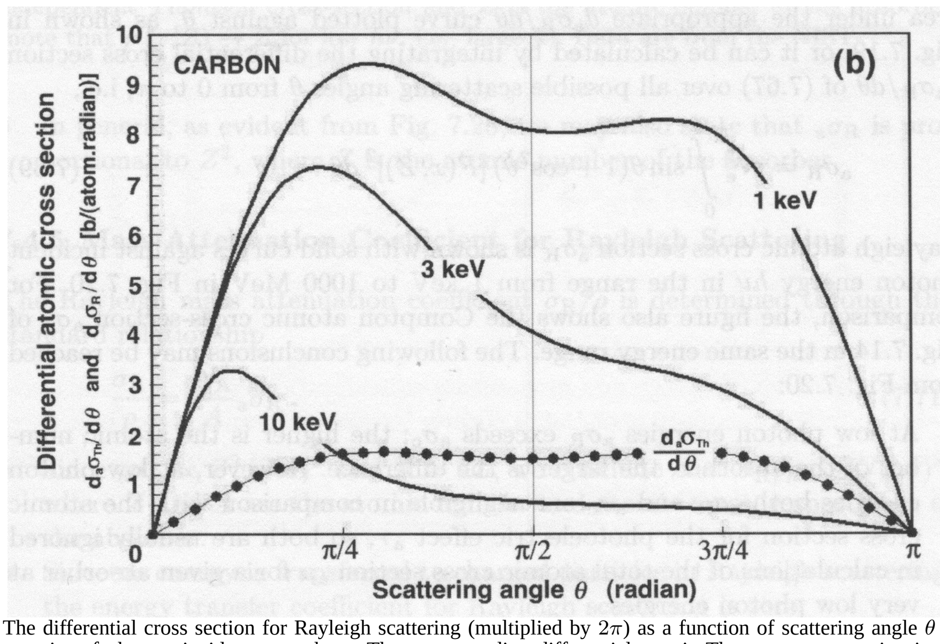
where

- $d_{\rm e}\sigma_{\rm T}/d\Omega = \frac{1}{2}r_{\rm e}^2(1+\cos^2\theta) =$ differential Thomson cross section, given by (13)
- $F(x,Z) =$ *atomic form factor*
- $x = \lambda^{-1}\sin(\theta/2) =$ momentum transfer variable
- $\lambda =$ incident photon wavelength
- $F(x,Z)$ determined by atomic wavefunctions, so generally complicated functions
- $F(x,Z) \approx Z$ for $\theta \approx 0$
- $F(x,Z) \approx 0$ for $\theta \to \pi$



The atomic form factor for Rayleigh scattering for different atomic numbers ranging from $Z = 1$ (H) to $Z = 82$ (Pb). (Fig. 7.18 in Podgoršak)

- $d_{\rm a}\sigma_{\rm R}/d\Omega$ becomes increasingly asymmetric about $\theta = \pi/2$ as $h\nu$ increases due to forward beaming



The differential cross section for Rayleigh scattering (multiplied by 2π) as a function of scattering angle θ for varying energies of photons incident on carbon. The corresponding differential atomic Thomson cross section is also shown. $d_{\rm a}\sigma_{\rm T}/d\Omega = 6 d_{\rm e}\sigma_{\rm T}/d\Omega$ for carbon. (Fig. 7.19b in Podgoršak)

- *characteristic angle for Rayleigh scattering*:

$$\theta_{\rm R} \approx 2\sin^{-1}\left(\frac{0.026 Z^{1/3}}{\varepsilon}\right) \tag{16}$$

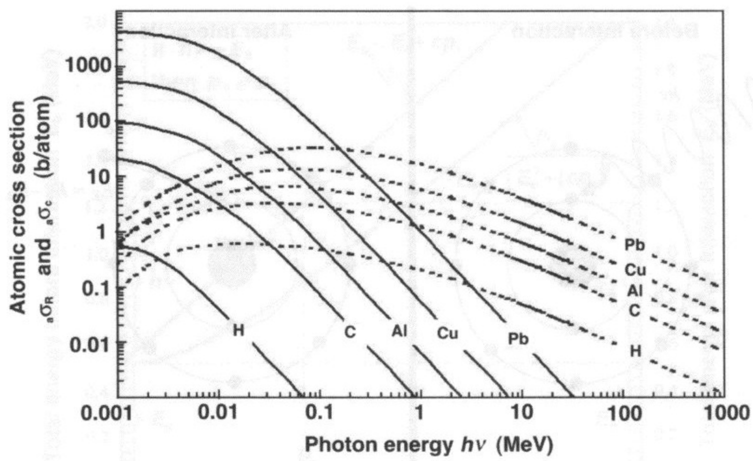
where $\varepsilon = h\nu/m_e c^2$

- opening half angle of cone containing 75% of scattered photons
- for high photon energies ($h\nu \gg 1\,\mathrm{MeV}$), scattering restricted to small angles
- *total atomic cross section*

$$_{a}\sigma_R = \pi r_e^2 \int_0^{\pi} \sin\theta(1+\cos^2\theta)\,\{F[x(\theta),Z]\}^2\; d\theta \tag{17}$$

- *mass attenuation coefficient*

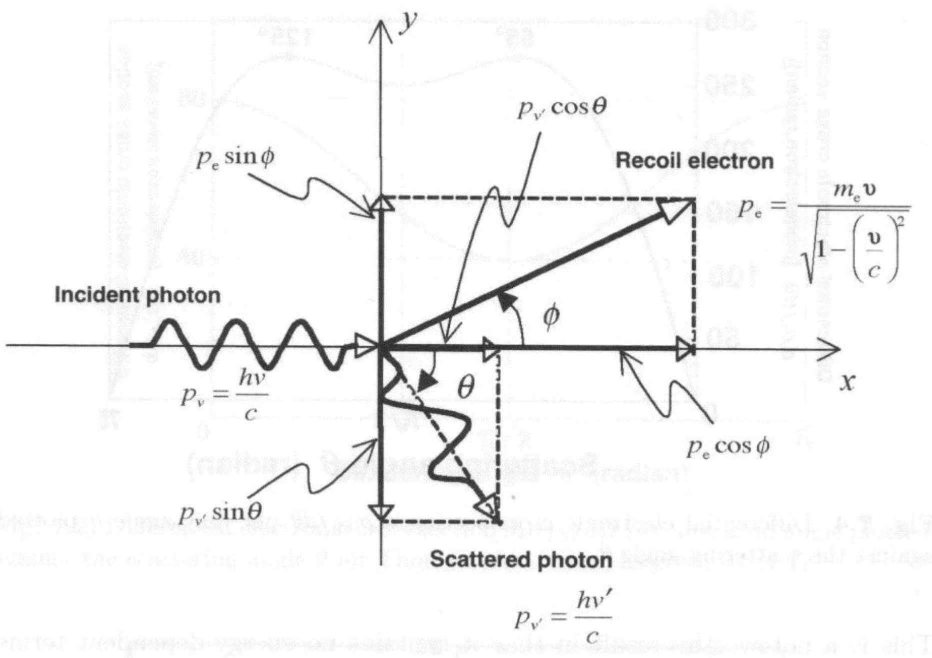
$$\frac{\sigma_R}{\rho} = \frac{N_A}{A}\,{}_{a}\sigma_R \tag{18}$$



The atomic cross section for Rayleigh scattering (solid curves) compared to that for Compton scattering (dotted curves) plotted against incident photon energy for varying Z atoms. (Fig. 7.20 in Podgoršak)

1.3.3 Compton Scattering

- *incoherent scattering* of photon off free (or loosely bound) electron
- *Compton recoil electron* ejected from atom with kinetic energy E_K



Kinematics:

$$h\nu = h\nu' + E_K \qquad \textit{energy conservation} \tag{19}$$

$$\mathbf{p}_\nu = \mathbf{p}'_\nu + \mathbf{p}_e \qquad \textit{momentum conservation} \tag{20}$$

where

- $E_K = (\gamma - 1)m_e c^2$, with $\gamma = (1 - \beta^2)^{-1/2}$
- $p_\nu = h\nu/c =$ initial photon momentum (prime denotes quantity after scattering)
- $p_e = \gamma\beta m_e c =$ momentum of recoil electron

Momentum parallel to initial photon propagation:

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + \gamma\beta m_e c \cos\phi$$

where $\phi =$ recoil angle (measured w.r.t. incident photon propagation) and $\theta =$ photon scattering angle. Momentum conservation normal to incident photon:

$$0 = -\gamma\beta m_e c \sin\phi + \frac{h\nu'}{c}\sin\theta$$

Solving simultaneously using identity $\gamma^2 - 1 = \gamma^2\beta^2$ gives

$$h\nu' = h\nu\left[1 + \varepsilon(1 - \cos\theta)\right]^{-1} \tag{21}$$

$$E_{\mathrm{K}} = h\nu\varepsilon(1-\cos\theta)\left[1+\varepsilon(1-\cos\theta)\right]^{-1} \tag{22}$$

where $\varepsilon = h\nu/m_{\mathrm{e}}c^2$.

- $E_{\mathrm{K}} \to 0$ as $\theta \to 0 \Rightarrow$ *Thomson limit*
- max energy transfer when $\theta = \pi \Rightarrow$ *backscatter*

In terms of photon wavelength:

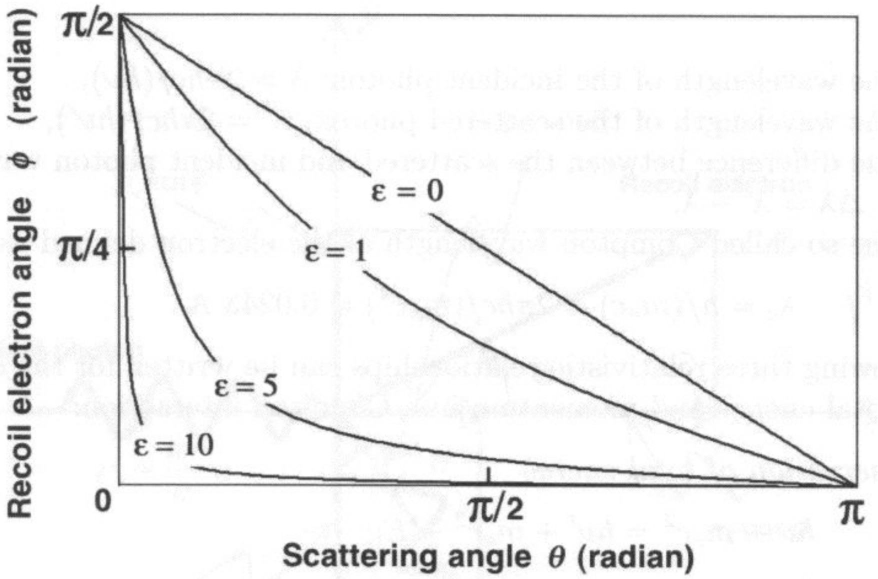
$$\Delta\lambda = \lambda' - \lambda = \lambda_{\mathrm{C}}(1-\cos\theta) \tag{23}$$

where $\lambda_{\mathrm{C}} = h/m_{\mathrm{e}}c \approx 0.0243\,\text{Å}$ is the *Compton wavelength*. So the *wavelength shift* is independent of incident photon energy. But the frequency shift is not.

- photon scattering angle θ and electron recoil angle ϕ are related by

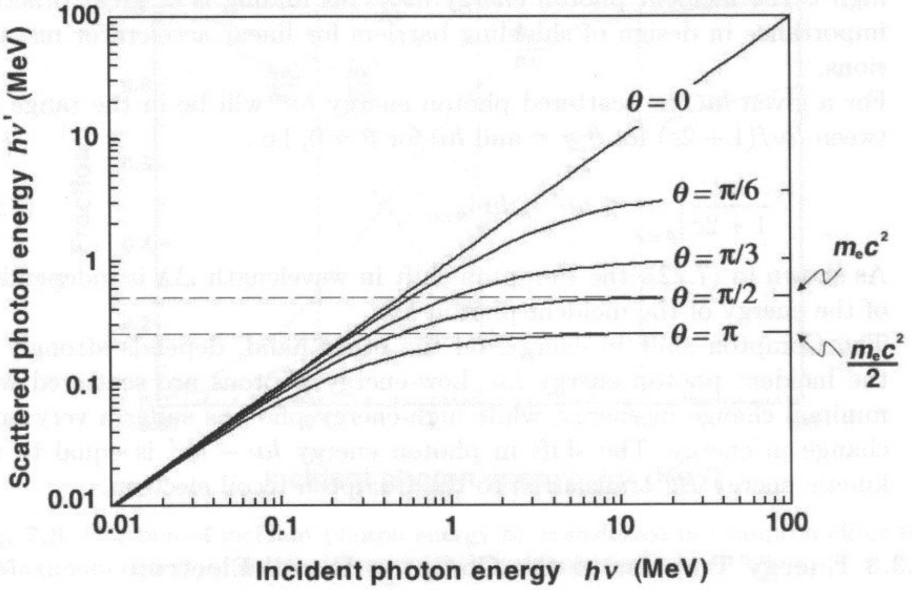
$$\cot\phi = (1+\varepsilon)\tan\left(\frac{\theta}{2}\right) \tag{24}$$

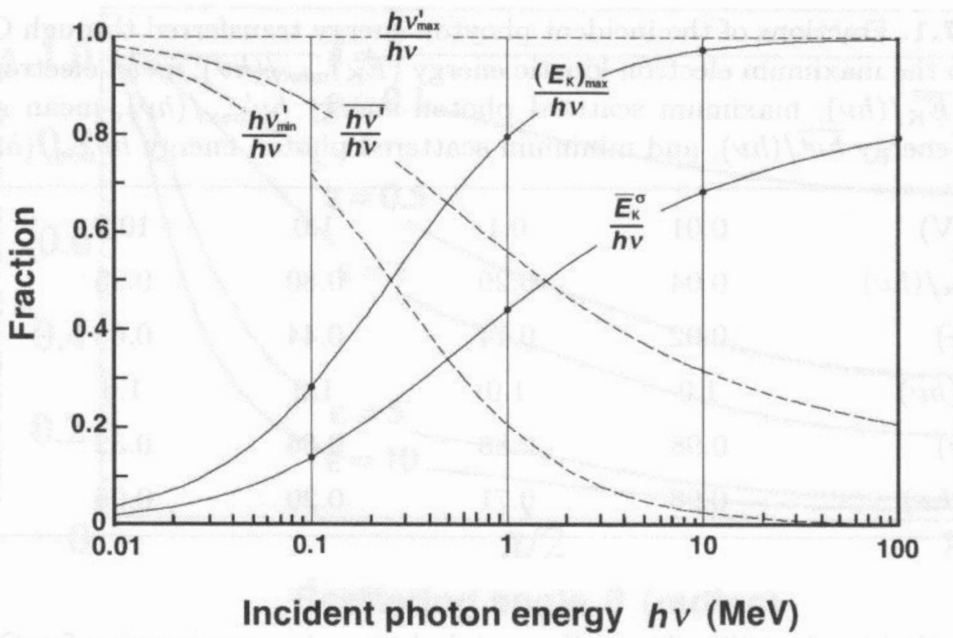
- recoil restricted to $0 \lesssim \phi \lesssim \pi/2$
- $\phi \approx 0$ for photon backscatter ($\theta \approx \pi$)



- for $\theta > 0$, $h\nu'$ saturates; $h\nu'$ cannot exceed $511\,\mathrm{keV}$ for $\theta > \pi/2$, regardless of $h\nu$

• high energy photons lose more energy





Fraction of incident photon energy $h\nu$ transferred to: maximum recoil energy $(E_{\rm K})_{\max}$ (for backscatter), mean recoil energy $\overline{E_{\rm K}^{\sigma}}$, maximum scattered photon energy $h\nu'$, mean scattered photon energy $\overline{h\nu'}$, and minimum scattered photon energy $h\nu'_{\min}$. (Fig. 7.8 in Podgoršak)

• photon backscattering gives maximum energy transfer to electrons; (22)

with $\theta = \pi$ implies

$$\frac{(E_\mathrm{K})_\mathrm{max}}{h\nu} = \frac{2\varepsilon}{1+2\varepsilon} \tag{25}$$

- range of recoil energies for other θ
- *mean recoil energy* $\overline{E_\mathrm{K}^\sigma}$ determines energy transfer cross-section for Compton scattering (to be discussed in next lecture)
- mean energy of scattered photons:

$$\frac{\overline{h\nu'}}{h\nu} = 1 - \frac{\overline{E_\mathrm{K}^\sigma}}{h\nu}$$

- minimum energy of scattered photons:

$$\frac{h\nu'_\mathrm{min}}{h\nu} = 1 - \frac{(E_\mathrm{K})_\mathrm{max}}{h\nu} = \frac{1}{1+2\varepsilon}$$