PHYS 5012
Radiation Physics and Dosimetry

Lecture 1

Tuesday 3 March 2009

Contents

1 Background and Fundamentals

Cleaning up Chernobyl: workers are advised to limit exposure to no more than 15 mins at a time (National Geographic Magazine, April 2006).

1.1 The Discovery of Radiation

Three main discoveries of radiation made at the turn of the 19th century, together with several major advances in theoretical physics, including quantum mechanics and special relativity, signalled the birth of Radiation Physics. The subsequent realisation that radiation can be harmful to humans led to the the rapid development of radiation dosage measurements and quantification and commonly accepted standards for tolerable levels of radiation in humans.

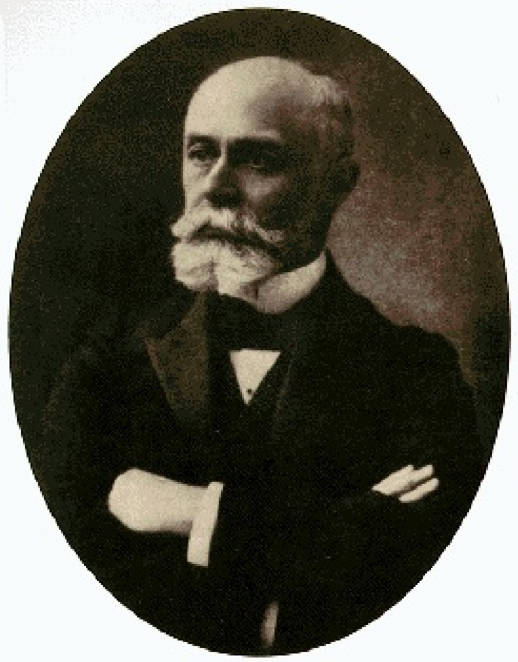
1.1.1 X-rays

X-rays are photons (i.e. electromagnetic radiation) with energies typically above 1 keV. They were discovered by Wilhelm Conrad Roentgen in 1895.

Roentgen discovered X-rays inadvertedly whilst studying fluoresence using a cathode ray tube. He explored the absorption properties of the rays in soft tissue and bone using his wife's hand (note the ring).

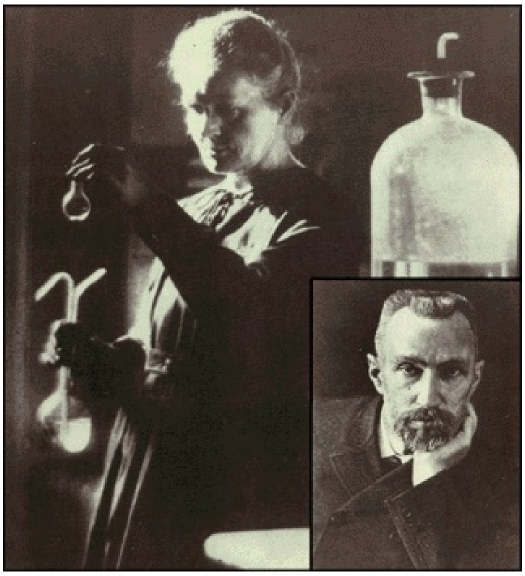
1.1.2 Radioactivity

Natural radioactivity is the spontaneous emission of radiation by a material. It was discovered by Antoine Henri Becquerel in 1896.



Whilst Roentgen's X-rays needed to be induced by cathode rays (electrons), Becquerel found that some materials, notably uranium ore, possessed their own source of radiation energy. He discovered this after placing some uranium mineral on a photographic plate wrapped in black paper into a dark drawer, finding afterwards that the uranium had indeed left an image on the plate.

Marie Curie coined the term "radioactivity" for the phenomenon Becquerel found associated with uranium ore. Together with her husband Pierre, they began investigating radioactivity. Marie found that after extracting pure uranium from ore, the residual material was even more radioactive than the uranium. She had discovered polonium and radium.



1.2 Classification of Radiation

Radiation can be broadly classified into two main categories, based on its ability to ionise matter:

- *Non-ionising radiation* cannot ionise matter because its energy is lower than the ionisation potential of the matter.
- *Ionising radiation* has sufficient energy to ionise matter either directly or indirectly.

Although non-ionising radiation can transfer some of its energy to matter, the low energies involved result in negligible effects compared to those of ionising radiation. Henceforth, only ionising radiation will be considered.

1.2.1 Types of Ionising Radiation

Ionising radiation can be further subdivided into two classes:

- *Directly ionising* - charged particles (electrons, protons, α particles, heavy ions); deposits energy in matter directly through Coulomb collisions with orbital electrons.

- *Indirectly ionising* - neutral particles (photons, neutrons); deposit energy indiectly through a two-step process: 1. release of charged particles and 2. charged particle energy deposition through Coloumb interactions.

Charged particles are described as **light** (electrons and positrons), **heavy** (protons, deutrons, α particles) or **heavier** (e.g. carbon-12). Some of the common nomenclature is as follows:

Light charged particles

- *photoelectrons* – produced by photoelectric effect
- *recoil electrons* – produced by Compton effect
- *delta rays* – electrons produced by charged particle collisions
- *beta particles* – electrons or positrons emitted from nuclei by β^- or β^+ decay: ${}^1_0n \longrightarrow^1_1 p +^0_{-1} e$ or ${}^1_1p \longrightarrow^1_0 n +^0_{+1} e + \nu$

Heavy charged particles

- *protons* – nucleus of hydrogen-1 (${}^1_1H$) atom
- *deutron* – nucleus of deuterium (${}^2_1H$) atom
- *triton* – nucleus of tritium (${}^3_1H$) atom
- *helium-3* – nucleus of helium-3 (${}^3_2He$) atom
- α *particle* – nucleus of helium-4 (${}^4_2He$) atom

Heavier charged particles include nuclei or ions of heavier atoms such as carbon-12 (${}^{12}_6C$), nitrogen-14 (${}^{14}_7N$), or neon-20 (${}^{20}_{10}Ne$).

Ionising photons can be classified into four groups:

- *characteristic X-rays* – due to electronic transitions between discrete atomic energy levels
- *bremsstrahlung emission* – due to electron-nucleus Coulomb interactions
- *gamma rays* – resulting from nuclear decays
- *annihilation radiation* – resulting from electron-positron pair annihilation

1.3 Radiation Units and Properties

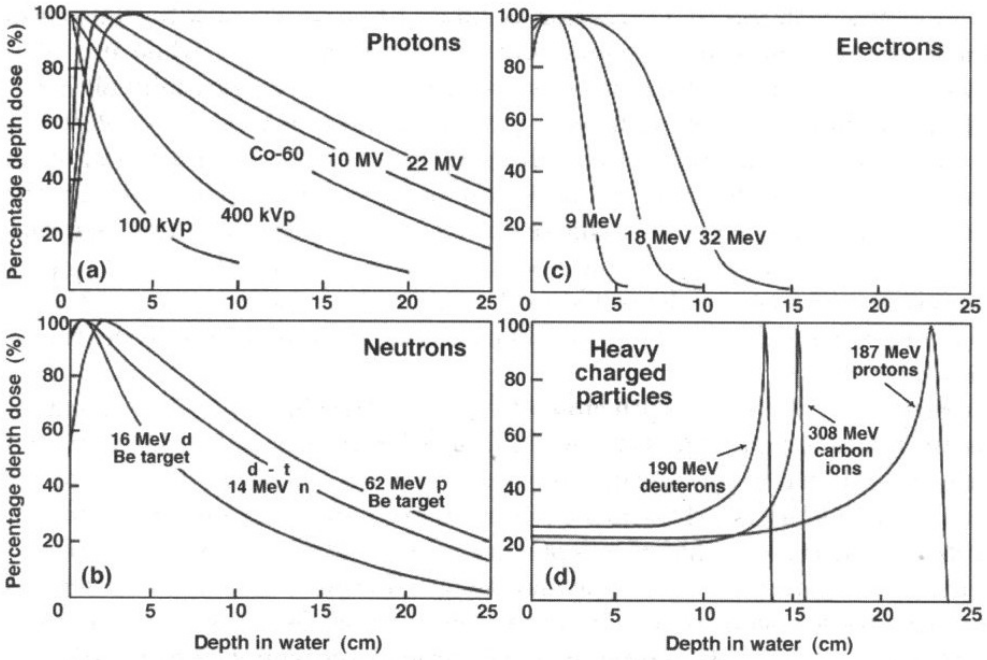
Accurate measurement of radiation is critical to any industry or profession that involves regular use of radiation. Several units have been defined to quantify different types of radiation measurements. These are summarised in the following table.

Quantity	Definition	SI unit
Exposure	$X = \Delta Q/\Delta m_{\mathrm{air}}$	$2.58 \times 10^{-4}\mathrm{C\,kg^{-1}}$
Dose	$D = \Delta E_{\mathrm{ab}}/\Delta m$	$1\,\mathrm{Gy} = 1\,\mathrm{J\,kg^{-1}}$
Equivalent dose	$H = Dw_{\mathrm{R}}$	$1\,\mathrm{Sv}$
Activity	$\mathcal{A} = \lambda N$	$1\,\mathrm{Bq} = 1\,\mathrm{s^{-1}}$

- *Exposure* measures the ability of photons to ionise air (its original unit of measurement was the roentgen, R); ΔQ is the collected charge.
- *Dose* is the energy absorbed per mass of matter; its unit is the gray (Gy); ΔE_{ab} is the energy absorbed in a medium.
- *Equivalent dose* is the dose mulitplied by a radiation weighting factor w_{R} for different types of radiation ($w_{\mathrm{R}} = 1$ for photons and electrons); its unit of measurement is the sievert (Sv).
- *Activity* is the number of decays per unit time of a radioactive substance; λ is the decay constant and N is the number of radioactive atoms.

1.3.1 Dose in Water

Dose deposition in water is extremely important because soft tissue is mostly made up of water. Different types of radiation deposit their energy at different depths in water. In general, indirectly ionising radiation deposits energy in an exponential-like fashion, while directly ionising radiation deposits virtually all its energy in a localised region, as is evident in the figure below.



Depth dose curves for different radiation beams in water and for different energies, normalised to 100% at depth dose maximum (reproduced from Podgoršak, Fig. 1.2).

Dose distributions for photon beams:

- *build-up region* from surface to depth dose maximum $z_{\max}$ followed by approximate *exponential attenuation*
- dose deposition determined by *secondary electrons*; $z_{\max}$ proportional to beam energy
- *skin sparing effect*: low surface dose for high energy beams

Dose distributions for neutron beams:

- similar to photon case, but dose deposition due to *secondary protons* or heavier nuclei

Dose distributions for electron beams:

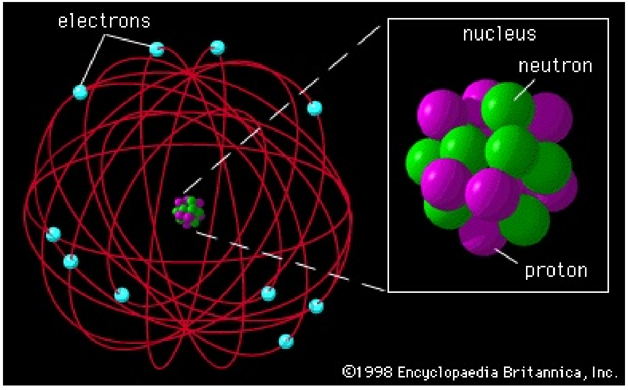
- *high surface dose* and build-up to $z_{\max}$, followed by rapid fall-off to a low-level dose *bremsstrahlung tail* due to radiative losses of the beam
- $z_{\max}$ does not depend on beam energy, but *beam penetration depends on beam energy*

Dose distributions for heavy charged particle beams:

- exhibit a *range in distance* traversed before *very localised energy deposition*; this is because of negligible changes in heavy particle trajectories resulting from Coulomb interactions with orbital electrons in absorber
- maximum dose is called *Bragg peak*

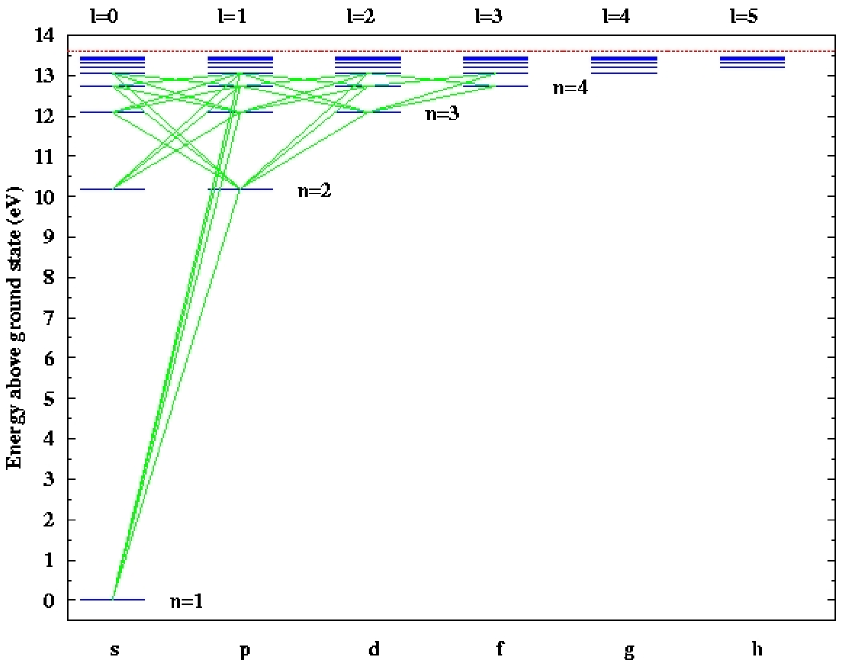
1.4 Atomic Physics and Radiation

With the discovery of electrons as well as alpha, beta and gamma rays by 1900 came their use as probes to study the atomic structure of matter. In 1911, Ernest Rutherford proposed the atomic model that we retain today, in which all positive charge is concentrated in a small massive nucleus, with the electrons orbiting around. This model was vindicated in 1913 by Rutherford's students, Geiger and Marsden, in their famous alpha particle scattering experiment (now known as "Rutherford scattering").



1.4.1 The Rutherford-Bohr Model

Neils Bohr further postulated that electrons only exist in certain fixed orbits that were related to the quantisation of electromagnetic radiation shown by Planck. Bohr's atomic model successfully explains single-electron atoms.

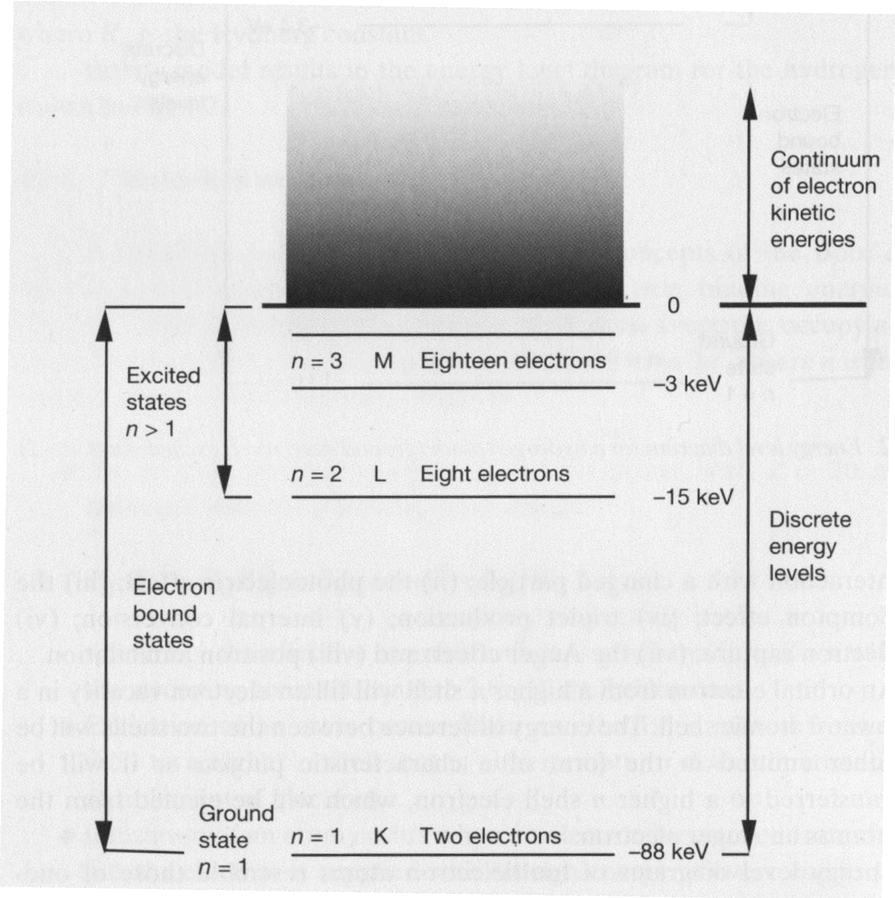


1.4.2 Multi-Electron Atoms

Bohr's model breaks down for multi-electron atoms because it does not take into account the repulsive Coulomb interactions between electrons. Douglas Hartree proposed an approximation that adequately predicts the energy levels E_n and radii r_n of atomic orbits in multi-electron systems:

$$E_n = -E_{\mathrm{R}} \left(\frac{Z_{\mathrm{eff}}}{n} \right)^2 \qquad , \qquad r_n = \frac{a_0 n^2}{Z_{\mathrm{eff}}} \tag{1}$$

where n is the *principal quantum number*, $E_{\mathrm{R}} = 13.61\,\mathrm{eV}$ is the *Rydberg energy*, Z_{eff} is the *effective atomic number* and $a_0 = 5.292 \times 10^{-11}\,\mathrm{m}$ is the *Bohr radius* of a single-electron atom.



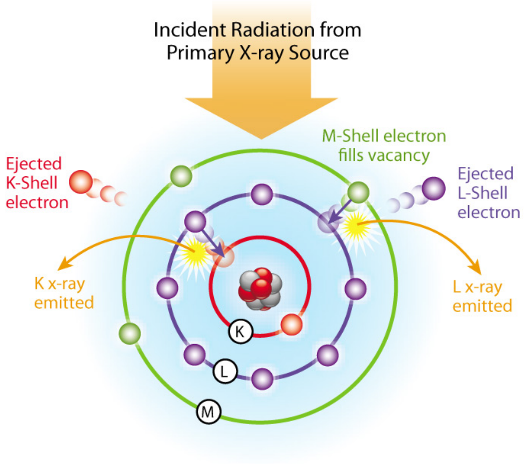
Energy level diagram for lead ($Z = 82$). The $n = 1, 2, 3, 4...$ shells in multi-electron atoms are referred to as the $K, L, M, N...$ shells.

2 Production of Radiation

Radiation is produced in a variety of different ways by both natural and man-made processes. Atoms in an excited state de-excite by emitting electromagnetic radiation at discrete energies. For high-Z atoms, this line emission typically occurs at X-ray energies and is referred to as *characteristic radiation*. Under some conditions, an excited atom can also de-excite by emitting an *Auger electron*, which is analogous to a photoelectron. Continuous emission of electromagnetic radiation is produced by charged particle (usually electron) acceleration, either by an electrostatic (Coulomb) field, resulting in *bremsstrahlung radiation*, or by a magnetic field, resulting in *synchrotron radiation*. Radiation can also be produced by naturally radioactive sources. This will not be covered here. Finally, man-made *accelerator machines* are designed to produce radiation with specific desired properties.

2.1 Characteristic Radiation

A vacancy in an atomic shell occurs as a result of several different processes (e.g. photoelectric effect, Coulomb interactions – to be discussed later in the course). When it occurs in an inner shell, the atom is in a highly excited state and returns to its ground state through *electronic transitions* which are usually accompanied by characteristic X-ray emission (formerly also referred to as *fluorescent emission*). Some transitions result in the ejection of other orbital electrons. This is the *Auger effect*.



2.1.1 Characteristic X-rays

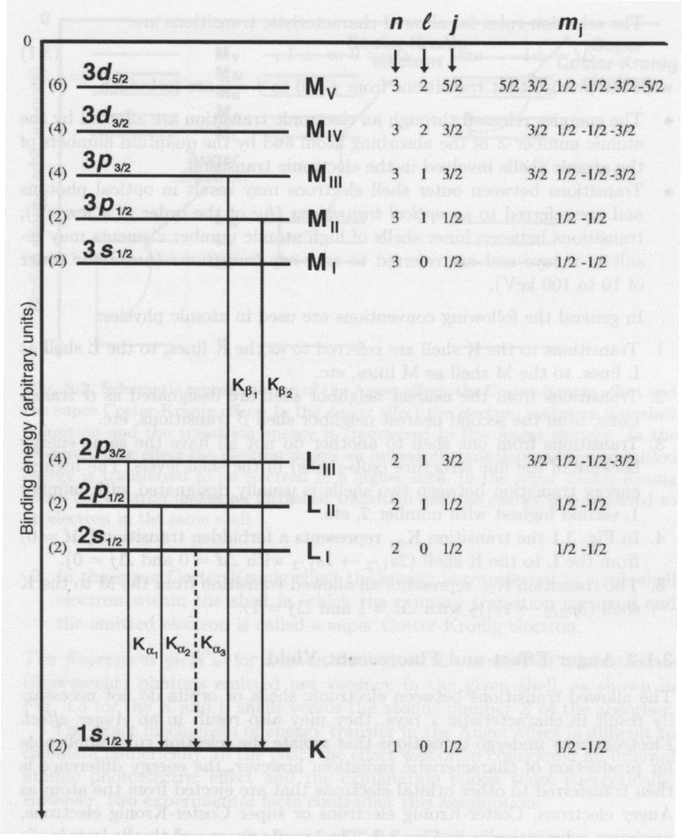
Electronic transitions that result in electromagnetic radiation are fully described using spectroscopic notation for the electronic configurations, which take the form nl_j written in terms of the quantum numbers:

- $n =$ ***principal quantum number***, or shell: $n = 1, 2, 3, ...$
- $l =$ ***azimuthal quantum number***, or subshell (specifying an electron's orbital angular momentum): $l = 0, 1, 2, 3, ..., n-1$ (corresponding to s, p, d, f orbital states)
- $s =$ ***spin quantum number***: $s = \frac{1}{2}$
- m_j = total (orbital+spin) angular momentum quantum number: $m_j = -j, -j+1, -j+2, ...j-2, j-1, j$, where $j = |l-s|, |l-s+1|, ...|l+s|$

Radiative transitions can only proceed between adjacent angular momentum states:

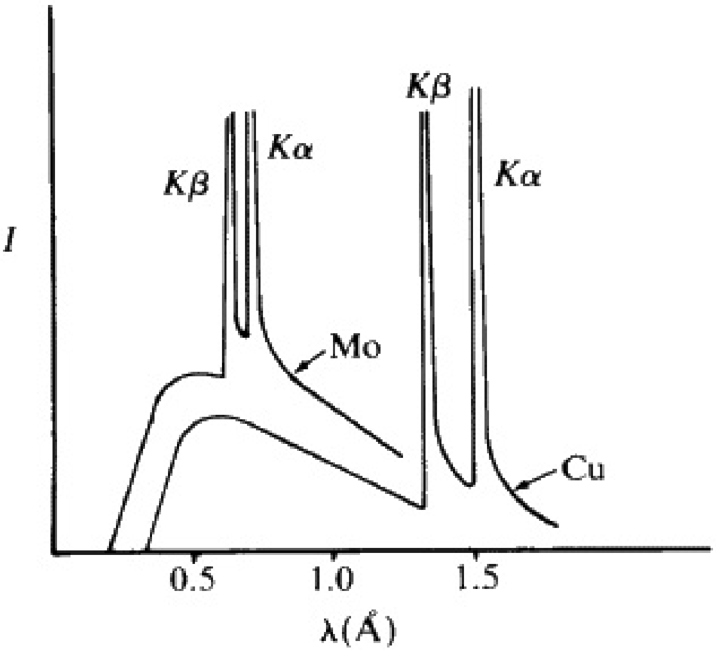
$$\Delta l = \pm 1 \qquad , \qquad \Delta j = 0 \text{ or } 1 \tag{2}$$

but not $j = 0 \rightarrow j = 0$. These are referred to as the *selection rules* for *allowed transitions* and are based on the condition that electrostatic interactions always dominate. *Forbidden transitions* are those which occur as a result of other interactions, the most important being spin-orbit (or $L - S$) coupling. Forbidden transitions violate the selection rules. For example, the K_{α_3} transition $2s_{1/2} \longrightarrow 1s_{1/2}$ is forbidden because $\Delta l = 0$ and $\Delta j = 0$. The K_{α_1} transition $2p_{3/2} \longrightarrow 1s_{1/2}$ is allowed because $\Delta l = 1$ and $\Delta j = 1$.



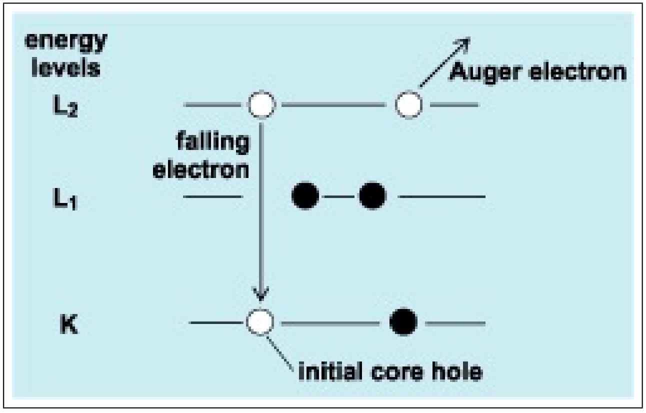
Typical energy level diagram for a high-Z atom showing sub-shell structure for K, L and M shells. Allowed (solid lines) and forbidden (dashed lines) K_α and K_β transitions are also shown. Numbers in parentheses indicate maximum number of electrons in that sub-shell, $2j + 1$. (From Podgoršak, Fig. 3.1.)

Characteristic emission produces *line spectra* at discrete energies corresponding to the difference between energy states. The strongest lines are usually the K_α ($n = 2 \rightarrow n = 1$) and K_β ($n = 3 \rightarrow n = 1$) transitions.



2.1.2 Auger Electrons

When forbidden transitions occur, sometimes it results in the ejection of an electron, called an *Auger electron*, instead of characteristic X-rays. The energy difference between the two shells is thus transferred to the Auger electron, which is ejected with kinetic energy equal to the difference between its binding energy and the energy released in the electronic transition. In the example shown below, for instance, the Auger electron's kinetic energy is: $E_{\text{kin}} = (E_K - E_{L_2}) - E_{L_2}$



The Auger effect usually occurs between L and K shells and is more common in low-Z atoms, which tend to have a lower *fluorescence yield* (number of characteristic photons emitted per vacancy) than high-Z atoms. This suggests the effect cannot be simply explained in terms of the photoelectric effect and photon reabsorption. In some cases, a cascade effect occurs, whereby inner shell vacancies are successively filled by the Auger process, with ejections of more loosely bound electrons. Atoms which produce mulitple Auger electrons are referred to as *Auger emitters*.

2.2 Continuous Radiation

Unbound charged particles that are accelerated emit electromagnetic radiation. The emitted photons can have any energy up to the kinetic energy of the radiating charged particle. Thus, the emission is continuous, rather than discrete as occurs for characteristic radiation. Emission of electromagnetic radiation is most efficient for electrons. The most common form of continous emission occurs when an electron is deaccelerated by the Coulomb field of a nearby atomic nucleus. This is called *bremsstrahlung radiation*. The radiation emitted by an electron accelerated by an external magnetic field is called *synchrotron radiation*. Radiative losses of high-energy particles are typically $\lesssim 10\%$.

The emission of electromagnetic radiation represents an irreversible flow of energy from a source (accelerated electron) to infinity. This is possible only because the electromagnetic fields associated with *accelerating* charges fall off as $1/r$, instead of $1/r^2$, as is the case for charges at rest or charges moving uniformly. This produces a finite total electromagnetic power ($\propto r^2E^2$) at arbitrarily far distances r. The $1/r$ dependence arises because electromagnetic waves have a finite propagation time to reach a field point P from a source point S, so the radiation field measured at P at time t depends on the time at emission, called the *retarded time*: $t' = t - \Delta r/c$.

The radiation field produced by an accelerated, nonrelativistic charge q is:

$$\mathbf{E}_{\text{rad}} = \frac{q}{4\pi\epsilon_0}\frac{1}{c^2}\left[\frac{\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \dot{\boldsymbol{v}})}{r}\right] \tag{3}$$

$$\mathbf{B}_{\text{rad}} = \frac{1}{c}\hat{\mathbf{r}} \times \mathbf{E}_{\text{rad}} \tag{4}$$

where $\dot{\boldsymbol{v}}$ is the particle's acceleration and $\mathbf{r}$ is the displacement vector from the charged particle at time $\boldsymbol{t'}$ to the field point at which the radiation is being measured at time $\boldsymbol{t}$. Note that $\mathbf{E_{rad}}$, $\mathbf{B_{rad}}$ and $\hat{\mathbf{r}}$ are mutually perpendicular.

The total electromagnetic power P radiated is obtained by integrating the Poynting flux, $S = E_{\text{rad}}B_{\text{rad}}/\mu_0$, over a surface area in all directions: $P = \int Sr^2 d\Omega$. This gives the following:

$$P = \frac{\mu_0 q^2 a^2}{6\pi c} \qquad \textit{Larmor formula} \tag{5}$$

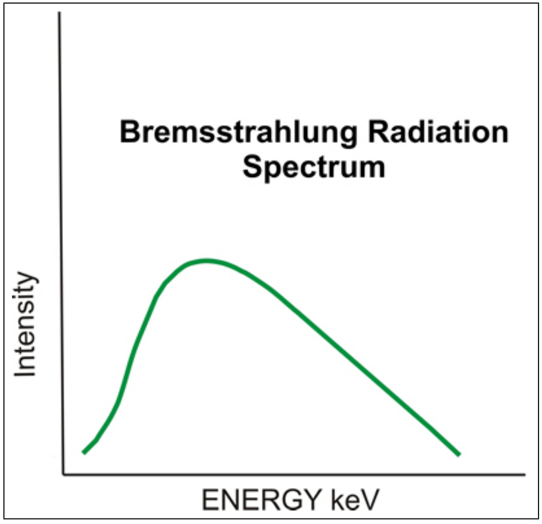
This famous result shows that the total power emitted into electromagnetic radiation is directly proportional to the square of a charged particle's acceleration a and charge q.

2.2.1 Bremsstrahlung Radiation

When charged particles of mass m and charge e are incident on a target material, they experience inelastic Coulomb interactions with the orbital electrons and with the nuclei (charge Ze) of the target. Coulomb collisions with the orbital electrons usually results in *ionisation losses*. Coulomb encounters with nuclei results in *radiative bremsstrahlung losses*. The acceleration a experienced by an incident charge in the vicinity of a nucleus is obtained from

$$ma = \frac{e\,Ze}{4\pi\varepsilon_0 r^2} \qquad \Longrightarrow \qquad a \propto \frac{Ze^2}{m}$$

The Larmor formula then implies that radiative losses for incident electrons will be more efficient, by a factor $(m_\mathrm{p}/m_\mathrm{e})^2 \simeq 4 \times 10^6$, than for protons, which will lose their kinetic energy more quickly via collisional ionisation losses.



The emission spectrum for bremsstrahlung radiation is continuous up to the kinetic energy of the emitting electrons. The spectrum peaks near the maximum kinetic energy, above which it declines rapidly.

2.2.2 Synchrotron Radiation

Synchrotron radiation is electromagnetic radiation emitted by charged particles accelerated by a magnetic field that maintains a circular particle trajectory, so there is a centripetal acceleration perpendicular to the instantaneous particle momentum. Because particles can be accelerated to very high energies, it is necessary to consider the relativistic generalisation of the Larmor formula:

$$P = \frac{\mu_0 q^2}{6\pi c}\gamma^4(\gamma^2 a_\parallel^2 + a_\perp^2) \qquad \textit{relativistic Larmor formula} \tag{6}$$

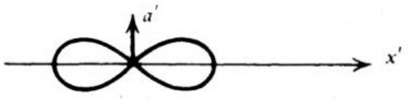
where $\gamma = (1-\beta^2)^{-1/2}$ is the particle's Lorentz factor, corresponding to its energy $E = \gamma mc^2$, and where $a_\parallel$ and $a_\perp$ are the components of the particle's acceleration parallel and perpendicular to its velocity $\boldsymbol{\beta}c$.

For synchrotron radiation, $a_\parallel = 0$ and $a_\perp = v^2/R$, where R is the fixed radius of the synchrotron accelerating device. The Larmor formula then implies

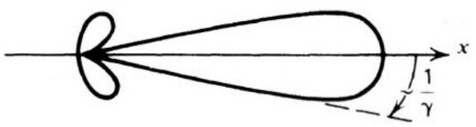
$$P = \frac{\mu_0 q^2 c^3 \beta^4 \gamma^4}{6\pi R^2} \tag{7}$$

For a fixed magnetic field strength B, the particle momentum attained is $\gamma mv = eBR$.

The radiation intensity pattern emitted by relativistic charged particles is highly directional and is beamed towards the direction of motion of the particles in a forward beam. This effect, called *relativistic beaming*, results from relativistic aberration.



dipole emission (particle rest frame)
$P(\theta) \propto \sin^2\theta$



forward beaming (observer rest frame)
$P(\theta) \propto (1-\beta\cos\vartheta)^{-4}$

Because $P \propto R^{-2}$ (c.f. eqn. 7), particle accelerators such as CERN's Large Hadronic Collider (LHC) and the Australian synchrotron (shown below) have to be built with a large radius of curvature in order to minimise synchrotron losses by the particles being accelerated.

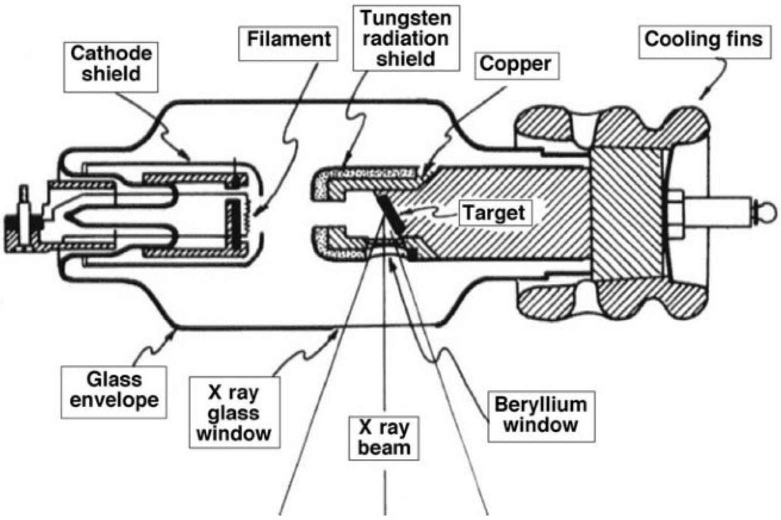
2.3 Particle Accelerators

Various types of particle accelerator machines have been built for basic research in nuclear and high-energy physics. Most of them have been modified for medical application. *All particle accelerators require an electric field to accelerate charged particles*. There are 2 types of electric field specifications:

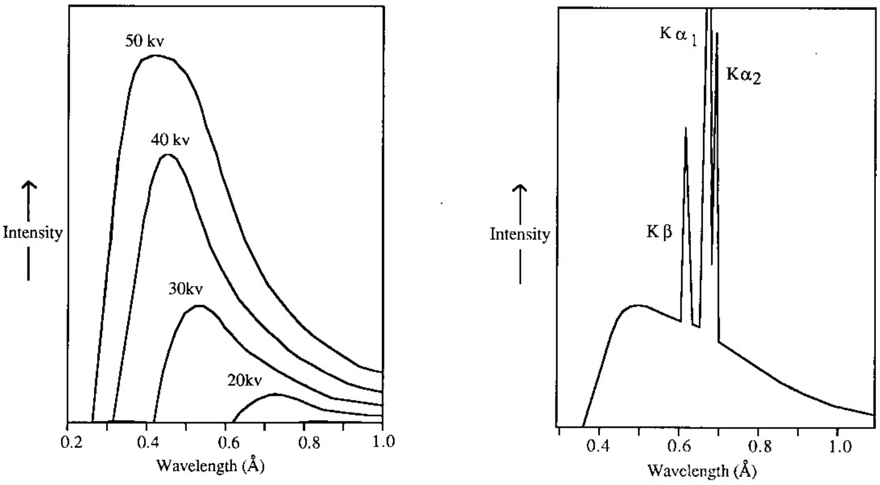
1. *electrostatic accelerators* – particles accelerated by a static electric field; maximum energy limited by voltage drop; examples: superficial and orthovoltage X-ray tubes.

2. *cyclic accelerators* – particles accelerated by time varying electric field and trajectories curved by associated magnetic field; multiple crossings of voltage drop allows high energies to be attained; examples: cyclotrons, linear accelerators.

2.3.1 X-ray Tubes

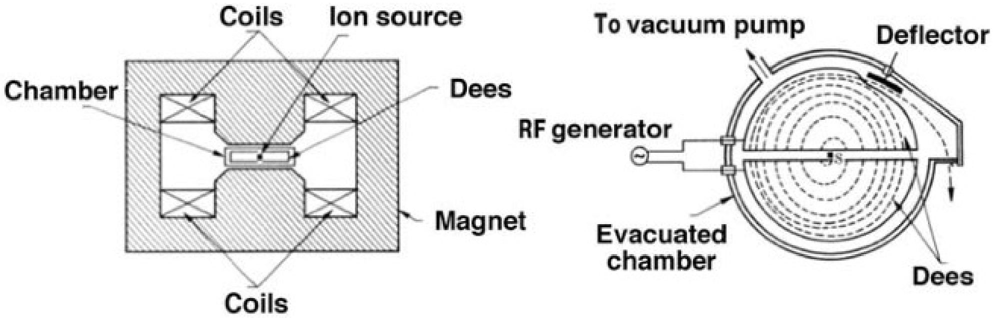


- electrons produced in heated filament (cathode) accelerated in vacuum tube toward target (anode) across electrostatic potential
- bremsstrahlung X-rays produced at high-Z target ($\sim$ 1% efficiency typically)
- kinetic energy deposited in target mostly as heat; requires cooling
- resulting X-ray beam energy determined by peak energy of electron beam (voltage drop), often given as peak voltage in kilovolts, kVp



Typical X-ray spectra produced by an X-ray tube. Left: bremsstrahlung only; Right: bremsstrahlung plus characteristic emission. ($1\,\text{Å} \approx 12.4\,\mathrm{keV}$)

2.3.2 Cyclotrons



- particles accelerated by crossing a radiofrequency (RF) voltage multiple times

- uniform B-field confines particle trajectories to spiral motion

- proton cyclotrons used to produce fluorine-18 radionuclide used in Positron Emission Tomography (PET)

2.3.3 Linear Accelerators

- used for radiotherapy treatment of cancer (external beam therapy)

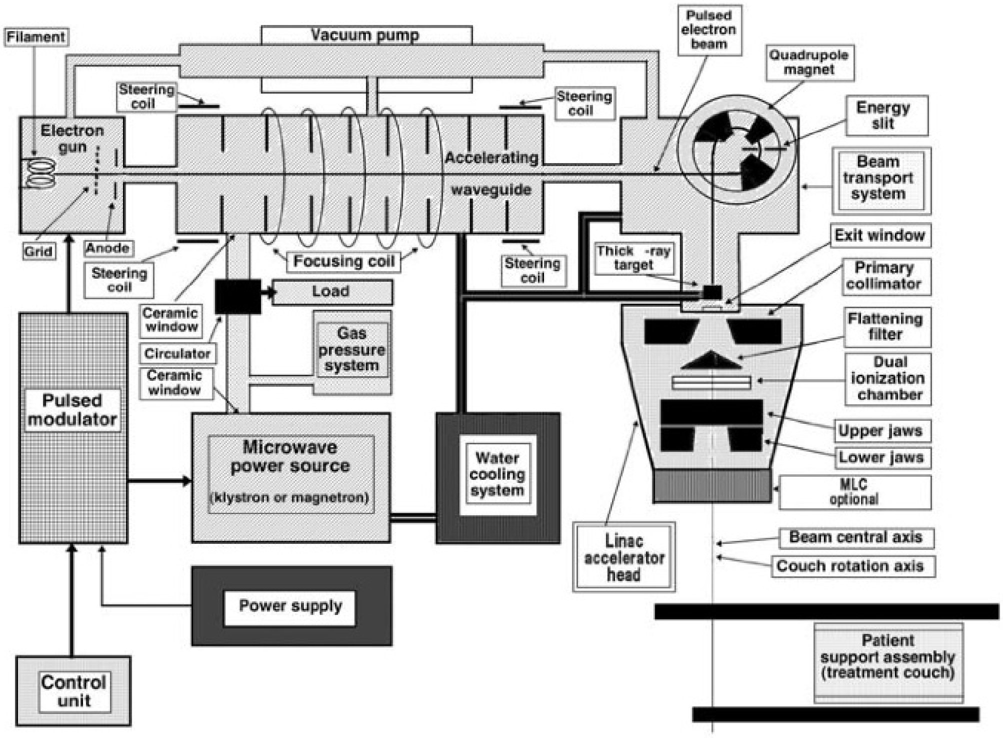
- acceleration of electrons by pulsed, high power RF fields in an *accelerating waveguide*

- linear trajectories, multiple voltage crossings

- peak electron beam energies in range $4 - 25\,\mathrm{MeV}$

- high energy ($5 - 20\,\mathrm{MeV}$) photon beams also produced with retractable thick X-ray target

- multiple configurations possible



Schematic of a medical linac (from Podgoršak, Fig. 3.10).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